

# Time to dump economic theory?<sup>1</sup>

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## Abstract

The current crisis is used as a pretext to present some of the models of market economies that have been developed in economic theory and to highlight properties of these models that illustrate several possible malfunctions of the market system.

## Resumen

La crisis actual se utiliza como pretexto para presentar algunos de los modelos de economías de mercado que se han desarrollado en la teoría económica y para destacar las propiedades de estos modelos que ilustran varios posibles fallos del sistema de mercado.

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## 1 Introduction

The main goal of this paper is to highlight some questions that have been addressed in economic theory and that may have some relation, possibly remote, with the current economic and financial crisis.

States have bailed out many banks and financial institutions. These massive interventions of the State has spread from the financial sector to

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the car industry and is now reaching other sectors as well. No one questions the fact that these interventions have kept alive the existing banking sector, at least in the short-run. Some economists argue, however, that State intervention is far from costless and is not necessarily the most efficient approach from a long-run perspective. There is therefore a renewed interest for the economic analysis of the State. The extent of the current crisis and the massive state interventions in many developed countries have given an unexpected new vitality to old criticisms of the capitalist system.

It has also become fashionable again to criticize the market system and to welcome a far more active role of the State. This new development comes after the worldwide trend to privatization and deregulation of the past three decades. When things go wrong, it is only natural to try to fix the cause. In the case of market economies, the only way to do that is through direct or indirect State intervention. If this explains the calls for a more active role of the State in the economy, we should not forget that heavy-handed State intervention is not necessarily efficient. And this is an understatement. The question is then: Can economic theory help us?.

## **2 Is economic theory trivial?**

A widely held view in the so-called hard sciences (physics and mathematics in particular) is that the soft sciences (all social sciences) have little to offer. Paul Samuelson tells us that he was once challenged by the mathematician Stanislaw Ulam to come up with a theory in the field of social sciences that would be true and not trivial. For mathematicians, a property is trivial when its proof is an obvious or almost obvious consequences of the assumptions. Mathematics is full of non trivial propositions. Does there exist at least one non trivial proposition in Economics?

Samuelson who is known for his legendary wit took several years before giving an answer to Ulam. This delay could be taken as a confirmation of the triviality of the social sciences. I would rather take it as evidence of the extreme care taken by Samuelson to come up with an example that would be beyond criticism. Samuelson's answer to Ulam is Ricardo's theory of

comparative advantage [30] of which he writes in [32] "That it is logically true need not be argued before a mathematician; that it is not trivial is attested by the thousands of important and intelligent men who have never been able to grasp the doctrine for themselves or to believe it after it was explained to them." (For details on the theory of comparative advantage, see for example [16].) The theory of comparative advantage is the cornerstone of the theory of international trade, already a theory of some kind of markets.

Ricardo's formulation of the theory of comparative advantage is not mathematical. Most textbook formulations of the theory follow closely Ricardo's own formulation and hide the underlying mathematical model. Nevertheless, the theory of comparative advantage deals with the properties of a mathematical model that represents production and trade between two countries. In fact, this mathematical model is the first serious model in economic theory, predating Cournot's models by more than 20 years [19]. The mathematical model that underlies the theory of comparative advantage is rather simple in the sense that it involves only linear equations. Despite its simplicity, some of its properties are not really obvious as Samuelson rightly points out.

### 3 General equilibrium theory

The mathematical model that underlies Ricardo's theory of comparative advantage is the first in a long list of more and more complex mathematical models of the economy. Walras' theory of general equilibrium is a far-reaching generalization of the simple linear model of comparative advantage [37, 38].

General equilibrium theory considers a collection of economic agents that meet in a market with the purpose of exchanging goods. These agents are endowed with resources that they bring to the market before the latter opens. Once the market opens, trade takes place at the so-called market prices, i.e., publicly posted prices. It is assumed that any arbitrary quantity of goods can be bought or sold at those prices. Each agent determines his

demand given his own wealth and market prices. Once each agent's demand is satisfied, i.e., once each agent holds the quantities of goods he wanted to get, the market closes. The general equilibrium model provides a set of equations that expresses the idea that the demand and supply of every commodity are equal in a competitive market.

Since there is no production in this version of the model, economic agents are called consumers. Let  $m$  be the number of consumers, and let  $i$ , with  $1 \leq i \leq m$  denote one such consumer. Let  $\ell$  denote the number of goods. We denote by  $\omega_i$  the vector representing the resources brought by consumer  $i$  to the market, and by  $x_i$  the bundle of goods brought back by consumer  $i$  after the market has closed. The idea is that  $x_i$  is used by consumer  $i$  for his consumption. Let  $\omega = (\omega_i)$  and  $x = (x_i)$  denote the collection of individual resources or endowments and individual consumption bundles. These endowment and consumption bundles belong to some set  $\Omega$ . To fix ideas, we can assume that  $\Omega = X^m$  where  $X = \mathbb{R}_{++}^\ell$  denotes the strictly positive orthant of the commodity space  $\mathbb{R}^\ell$ . This is to avoid the possibility of negative consumption for example.

The theory of general equilibrium provides us with an equation system

$$A(x, \omega) = 0$$

where the parameters of the equation system is the endowment vector  $\omega \in \Omega$  and the unknown the consumption bundle  $x \in \Omega$ . The solution  $x \in \Omega$  is often known as an equilibrium allocation associated with the endowment vector  $\omega \in \Omega$ . Note that the parameter  $\omega$  and the solution  $x$  belong to the same set  $\Omega$ .

The question is then: What are the properties of this equation system? More accurately, what is it that can be said of the properties of the solutions  $x \in \Omega$  of this equation system for  $\omega \in \Omega$ .

## 4 Existence and extremality of solutions

During the period 1874-1970, the two major achievements of the theory of general equilibrium were: 1) the proof of the existence of a solution under

reasonably realistic assumptions regarding consumer's behavior; 2) the definition of an extremality property for the elements of  $\Omega$  known as Pareto efficiency and the proof again under reasonably realistic assumptions that solutions satisfied the extremality property. These "reasonably realistic" assumptions evolved towards more and more generality during that time interval to become by the end of the 1940s formulated in a way that Nicolas Bourbaki, the mythical author of the monumental treatise of modern mathematics, would not have disapproved of.

Incidentally, the extremality property (2) has a converse, namely that every Pareto efficient allocation can be realized as the equilibrium allocation of some endowment vector  $\omega \in \Omega$ . Property (2) and its converse are known by economists as the two theorems of welfare economics. This extremality property is the most rigorous formulation of an idea that has been pervasive among many economists since the early XIXth Century, namely the superiority of free markets over alternative forms of resource allocation. On the mathematical side, this property can be compared with the principle of least action for mechanical systems [41]. Unsurprisingly, its importance in economic theory is comparable to the importance of the principle of least action in mechanics.

Existence was proved independently by Arrow and Debreu in [4] and McKenzie [29]. The two theorems of welfare economics were proved independently by Arrow [1] and Debreu [20]. Had Nicolas Bourbaki shown some interest for Economics and decided to devote one section of his treatise to Economics, he would have reformulated the Walrasian system of equations in a way not very different from Debreu's in [21]. This reformulation of the Walrasian system of equations is often known as the Arrow-Debreu model. The equilibrium equation in that model is not differentiable. Continuity and convexity (of preferences) suffice for the properties of existence and the theorems of welfare economics to hold true.

Despite their interest, the two properties of existence and the theorems of welfare economics do not tell us much about the important question of the dependence of the equilibrium solutions  $x \in \Omega$  on the parameters  $\omega \in \Omega$  of the equilibrium equation  $A(x, \omega) = 0$ , a topic known in economics as comparative statics. This easily explains that some economists got frus-

trated with a theory of general equilibrium that in 70 years had been able to prove only existence and the two theorems of welfare economics, frustration voiced vehemently and somewhat unfairly by the economist Mark Blaug [15].

## 5 Comparative statics

The equation system of the theory of general equilibrium has far more properties than just existence and the two theorems of welfare economics and these properties will be particularly interesting for comparative statics. But a prerequisite for addressing these new properties is to restrict the wide generality of the assumptions used in the Arrow-Debreu model. It is in fact by some kind of mathematical miracle that existence requires nothing more than continuity. This is because Brouwer's fixed point theorem [40], the prototype of all fixed point theorems, workhorses for proving the existence of solutions to general equation systems, requires only continuity. But it is almost impossible to go beyond existence without some form of differentiability. For example, comparative statics has a local version corresponding to small changes of the economic fundamentals and mathematicians will recognize in such issues the setup of the implicit function theorem, which requires differentiability.

### 5.1 Number and continuity of solutions

Under suitable assumptions about preferences, the equilibrium equation

$$A(x, \omega) = 0$$

is differentiable [23]. A remarkable result due to Debreu in 1970 is the proof that for almost all values of the parameter  $\omega \in \Omega$ , the set of solutions is finite [22]. In practice, this means that for  $\omega$  picked at random, the number of solution is finite with probability one. In addition, for such values of the parameter  $\omega$ , a small perturbation of the parameter induces only a small perturbation of the solution set, a property known as structural stability.

The importance of the concept of structural stability had been highlighted by the mathematician René Thom in relation with Catastrophe Theory that he developed in the late 1960s [36, 39]. The focus in Catastrophe Theory is placed on the values of the parameters for which the solution set is not structurally stable, i.e., not robust to small perturbations. Solutions are at such values discontinuous functions of the parameters. Thom gave the name of catastrophes to such discontinuities. Debreu’s main result is that the equilibrium of a market economy is a continuous function of the parameters defining the economy when parameters are chosen at random.

## **5.2 Uniqueness implies continuity**

If the solution of an equation is unique and stays unique when the parameters defining that equation are varied, there is no room for catastrophes. But the converse is not true. Structural stability does not imply uniqueness of the solution. Indeed, it suffices that each solution is a continuous function of the parameters for the collection of all solutions to be structurally stable.

## **5.3 Multiplicity implies discontinuity**

The economists Auspitz and Lieben recognized many years before Thom’s Catastrophe theory that the existence of multiple solutions can lead to catastrophes (in Thom’s sense) [6]. They did not address seriously this issue for lack of the proper mathematical tools. In their book, they assume away that possibility without realizing that this made their assumptions contradictory. Their continued interest for the multiplicity question is illustrated by the article that they published twenty year later without making any significant progress [7]. Fifteen years after that article, Bowley acknowledges in [17] that “there is nothing in the nature of the case to prevent multiple solutions . . .” but adds that “. . . in practice if we had any numerical values there is not likely to be difficulty in knowing which set is appropriate”.

Schumpeter is the first to fully realize in the late 1940s the importance of

the multiplicity issue when he writes “Multiple equilibria are not necessarily useless but . . . the existence of a ‘uniquely determined equilibrium. . .’ is, of course, of the utmost importance. . .” [33], p. 969. Schumpeter predates the literature on chaos and non-linear dynamics by more than two decades when he adds “. . . without any possibility of proving the existence of uniquely determined equilibrium—or at all events, of a small number of possible equilibria—at however high a level of abstraction, a field of phenomena is really a chaos that is not under analytic control”.

#### 5.4 Trade intensity and the number of equilibria

The question now is whether it is possible to extract from the properties of the general equilibrium model anything useful about the uniqueness of the solution of the equilibrium equation.

Starting with Walras, many economists thought that there would exist a set of assumptions that would be sufficiently general to be satisfied in most cases such that the equilibrium equation would have a unique solution. Walras believed that it would be sufficient to have more than two goods. Walras was wrong. In the 1930s, Wald showed that the solution of the equilibrium equation is unique if all goods are gross-substitute. This condition means that the demand of any good increases with the price of any of the other goods. This condition, nevertheless, is very restrictive. In practice, it is only satisfied by economies where consumers’ preferences are defined by log-linear utility functions. Wald’s gross-substitutability condition was subsequently weakened, but never sufficiently to be taken seriously as an indication that real world economies have a unique equilibrium.

In fact, in a version of the general equilibrium model that is slightly generalized by allowing some components of the endowment vector  $\omega = (\omega_i)$  to be negative, the uniqueness of equilibrium is easily seen to be equivalent to the constancy of the equilibrium price vector for all endowment vectors  $\omega$ . Clearly, this property is far from being satisfied in real economies.

It is therefore not realistic to expect that equilibrium is unique for all endowment vectors  $\omega \in \Omega$ . The uniqueness question then becomes the characterization of the subset  $\Omega_1$  of the parameter space  $\Omega$  that consists of



the endowment vectors such that the associated equilibrium is unique.

The main property in that direction is that the set of regular economies with a unique equilibrium  $\Omega_1$  is open and contains the set of equilibrium allocations  $P$ . In fact, this picture can be improved by observing that the set of equilibrium allocations  $P$  being connected, it is contained in one connected component of the open set  $\Omega_1$ . This property may sound quite abstract. It has a very concrete interpretation.

A sufficient condition for the endowment vector  $\omega = (\omega_i)$  to belong to  $\Omega_1$  (or, even more accurately, to the connected component of  $\Omega_1$  containing the set of equilibrium allocations  $P$ ) is that the vector  $x - \omega \in \Omega$  is sufficiently small. In other words, if the intensity of net trade at equilibrium is small enough, equilibrium is unique.

When the endowment vector  $\omega = (\omega_i)$  varies within the connected component  $\Omega_1$ , the unique equilibrium solution is a continuous (in fact differentiable) function of the endowment vector  $\omega$ . There are no discontinuities in the equilibrium allocation, no catastrophes in the sense of Thom.

The assumption of the uniqueness of equilibrium, implicitly or even explicitly as in [6], is widespread in the economic literature. The economic properties proved under these assumptions are at best, when reformulated within the proper general equilibrium setup, properties that may be satisfied only for an endowment vector  $\omega = (\omega_i)$  that belongs to the component  $\Omega_1$ . In other words, these properties are satisfied only for economic setups where the intensity of net trade is not too large. This remark applies in particular when it comes to the benefits of trade. To sum up, the arguments in favor of free trade that, starting with Ricardo's theory of comparative advantage, have been developed during the past two centuries, these arguments have a chance of remaining true within the general equilibrium setup only if the intensity of trade stays below some threshold value.

The question is then: What is it that can happen if the intensity of trade becomes large? For sufficiently large net trade vectors, the endowment vector  $\omega$  generally does not belong to the connected component  $\Omega_1$ . There are multiple equilibria associated with the endowment vector  $\omega$  even if only one is actually observed. Then, when the endowment vector is varied, it may very well cross the set of singular economies, the locus of these

catastrophes or discontinuities of the equilibrium selection process.

Interestingly, one consequence of international trade is the specialization of each country. Translated in terms of models, this means that the endowment vector  $\omega$  is moving away from the set of equilibrium allocations  $P$ . Therefore, a logical consequence of globalization is that the endowment vector  $\omega$  is moving within the domain of economies with multiple equilibria and, in its evolution through time, the endowment vector is likely to cross singular economies with the consequence that when this happens, the equilibrium may suddenly change from one value to a different one.

Such discontinuities may seem to lack real causes. Indeed, the change in the fundamentals can be sufficiently small not be noticeable. It triggers however a discontinuous change of equilibrium solutions.

## 5.5 Where to go from there?

These discontinuities of the equilibrium selection process are certainly not ideal. They perturb the formation of expectations of future prices. Every one remembers these pundits that were telling us in the Summer 2008 when the barrel of oil reached a price of US \$150 that it would be worth at least \$200 by January or February 2009. As of this writing, the price hovers around \$40.

Obviously, it would be helpful to be able to prevent these discontinuities of equilibrium selections. This is certainly one of the goals of the economic policies followed by many governments. But is it realistic to address this question with a view towards a practical solution within the simple setup of the general equilibrium model? Probably not. The general equilibrium model is already considered by many economists to be too complex. Nevertheless, it lacks many features. For example, there is no money. There is no financial sector either. Therefore, this simple model is very useful in calling our attention on the possibility and even the likelihood of discontinuities of equilibrium selections as a consequence of globalization; it is not rich enough, however, to enable us to come up with a workable cure to these discontinuities.

## 6 Production, the financial and monetary sector, and the State

It is obvious that if we want to generalize the pure exchange model so that it accounts for the most important forms of economic activity, it is necessary to have production, a financial and monetary sector, and the State as its own entity.

### 6.1 Production

The standard extension to production considered in the Arrow-Debreu model corresponds to private ownership. Firms are represented by way of their production set, the set of all technologically feasible activities. Firms maximize their profit subject to technological feasibility. These profits are then distributed to the owners. This is the model considered in Debreu's book [21]. Roughly speaking, this model has the same properties as the pure exchange model.

### 6.2 Time and uncertainty

Money and more generally assets are tools that enable us to transfer wealth through time. With the introduction of time comes uncertainty. Therefore, the introduction of a financial and monetary sector in the general equilibrium model requires us to introduce time and uncertainty in a model that is timeless in the previous section in the sense that time does not play any explicit role.

The simplest version of a general equilibrium model with time and uncertainty consists of two time periods, today and tomorrow, with tomorrow's uncertainty represented by a number of states of nature. This model is first described by Arrow in 1953 [2, 3]. Assets are characterized in terms of their payoffs in the various states of nature that may occur tomorrow. Money is just one asset among the others, one unit of money today promising to pay or to be worth one unit of money in tomorrow's states of nature.

In this model, assets and goods have very different roles. Goods are arguments of consumers' utility functions. Assets are not. In addition there are two kinds of assets. Real assets have payoffs consisting of goods. Purely financial assets have payoffs denominated in units of accounts or, more simply, in terms of the asset identified to money. The vast majority of assets in the real world are best described by the purely financial assets.

A remarkable property of the model with purely financial assets occurs when the asset market is incomplete. Market incompleteness means that there are more states of nature than there are assets. In such a case, uncertainty is defined by so many states of nature that economic agents are unable to hedge tomorrow's uncertainty. Mathematically, this takes the form of consumers maximizing their preferences over several budget constraints instead of the unique budget constraint of the Arrow-Debreu model.

The consequence is that the set of solutions of the equilibrium equation in the model with incomplete financial markets is not discrete. In other words, there is a continuum of solutions. Another remarkable property of this model is that the financial and real spheres are not independent from each other. The famous dichotomy property about the independence of the financial and real spheres that some economists claim to be true is satisfied only if asset markets are complete, a property that is not defensible in the asset model.

### **6.3 The State**

The State as an economic actor owns firms and redistributes resources among economic agents through fiscal and monetary policies. The general equilibrium model and its infinite horizon extension, the overlapping-generations model, easily accommodate such extensions. A very remarkable property shown by these models, a property particularly relevant in the current crisis, is that equilibrium is compatible with the State running a deficit. Samuelson is the first to provide such a remarkable example in [31]. In the overlapping-generations model, equilibria still satisfy an extremality property known as weak Pareto optimality in the sense that such alloca-

tion cannot be improved through redistribution of resources among a finite number of economic agents. Strong Pareto optimality corresponds to the standard version of Pareto efficiency in the sense that a strong Pareto optimum cannot be improved through the redistribution of resources among a possibly infinite number of economic agents. Samuelson's example is remarkable by the property that a non-trivial monetary equilibrium does not only exist but is also a strong Pareto optimum. For a while, it was even thought by some macroeconomists that public deficits were conducive to (strong) Pareto efficiency. The thorough analysis of efficiency within the overlapping-generations model combined with the characterization of public deficits compatible with the existence of non-trivial monetary equilibria made by Shell and me in [12, 13] shows us that public deficits are not sufficient to induce strong Pareto efficiency. But they are certainly a way to reallocate resources so that the resulting equilibrium allocations are strongly Pareto efficient.

## 7 Other extensions of the standard general equilibrium model

Instead of having time reduced to two or an infinite number of periods, there is room for a finite (possibly large) number of such time periods. Such models avoid the technicalities created by an infinite horizon and their results are certainly more amenable to interpretation than those for the two period model.

The general equilibrium model with a finite number of time periods lends itself remarkably well to the analysis of the credit crunch. In this model, some agents are prevented from borrowing money and transferring their wealth between time periods. Some equilibria are efficient, but some others may fail to be. In the stationary version of that model, a version aimed at studying the impact of economic fluctuations, I show for example that the credit crunch can yield equilibria that are inefficient [10].

The Cass-Shell sunspot model [18] is another example of a model where some consumers cannot transfer wealth between states of nature. A sunspot

equilibrium is one where the equilibrium allocation depends on the realization of a sunspot despite the fact that sunspots are assumed to have no impact on the fundamentals of the economy. Unsurprisingly, sunspot equilibria are again inefficient. Economies with sunspot equilibria exist as soon as some consumers cannot insure themselves against sunspots [11].

## 8 The temporary equilibrium model

We have seen that addressing sensitive economic issues required us to introduce time in our models. In the overlapping-generations model with an infinite number of time periods, the simpler models eliminate uncertainty. The equilibrium concept is then known as a perfect foresight equilibrium. At such equilibrium, aggregate demand and supply are equal at all time periods. Even if this may not seem too obvious at first sight, this concept is already interesting when it comes to understanding the fundamentals of economic dynamics. Introducing uncertainty by way of states of nature adds a touch of mathematical complexity that turns out not to alter significantly the nature of the properties of the models. But the way this extension is usually done introduces what is in my opinion a significant dose of economic irrelevance. In the real world, we do not have markets for the goods delivered at dates far away in the future. In practice, economic decisions made today cannot be based on prices determined on competitive markets. Economic agents have to substitute to these non existing market prices their own forecast of these prices.

This leads us to the temporary equilibrium model. The formal structure is identical to the one of two-period model with uncertainty in the second time period. But at variance with that model, there are no markets for the goods delivered in the second period. Instead, economic agents forecast the prices and quantities produced, supplied and demanded in the second period given the economic activity of the first period. The only markets that are open are for the goods delivered in the first period. Equilibrium prices equate the supply and demand of these goods. Obviously, supply and demand depend on the agents' forecasts of future prices.

The temporary equilibrium model was introduced by Lindahl and Hicks in the late 1930s [27, 28]. The model was reformulated with the modern standards of rigor by Arrow and Hahn [5] and independently by Stigum [34, 35] in the late 1960s, quickly followed by further work by Grandmont and Green [24, 25] among others. In these formulations, forecasts or expectations are arbitrary.

## 8.1 Rational expectations

By definition, expectations are rational when economic agents have a correct model of the economy, model that they apply to forecast the prices and economic variables that will be determined in the future markets when the latter will be open. The temporary equilibrium model with rational expectations is equivalent to the two period Arrow-Debreu model [8].

Unfortunately, economic agents are very far from having models of these future markets on which they would be able to base their forecasts. Rational expectation is for the moment nothing more than a valuable and helpful philosophical concept.

## 8.2 Equivalence between the temporary equilibrium model and the Arrow-Debreu model

Now, what models do we have for the markets open in the sense of the temporary equilibrium model? I showed a few years ago that a suitable formulation of the states of nature representing the uncertainty of the second period enables one to identify the temporary equilibrium model with a standard Arrow-Debreu model where economic agents have demand functions that depend on market prices and individual wealths [9]. The difference with the standard version of the Arrow-Debreu model is that these demand functions are not derived any more from the maximization of some preference relation subject to a unique constraint. In other words, these demand functions do not satisfy the restrictions that are lengthily studied in consumer theory, namely the symmetry and the negative definiteness of the Slutsky matrix associated with every demand function.

Now, in recent developments, Mich Tvede and I showed that these properties of the Slutsky matrix are not essential for the main properties of the Arrow-Debreu model to be satisfied [14]. Symmetry is in fact irrelevant. The Healy important property is the negative quasi-definiteness of the Slutsky matrix because, if satisfied for every consumer, the Arrow-Debreu model has the same properties as when consumers are preference maximizers.

An open but important problem is whether real world data confirm the negative quasi-definiteness of these individual Slutsky matrices. There is already a huge literature on this question for ordinary consumption goods. Future research will try to adapt the methodology developed for consumption goods to include financial assets.

At the moment, we can expect that the properties of the temporary equilibrium model are going to be those of a standard Arrow-Debreu model. The study of how the extensions of the Arrow-Debreu model to time and uncertainty like in the two-period model, the more general inter temporal model, the sunspot model and the overlapping-generations model fare under these more general assumptions regarding consumer's demand functions will be the subject of future research.

Paoli Zerilli of the Economics Department has recently applied to ESRC for a research grant that will investigate this question. The practical impact of this research is going to be even more important. The formulation of a computable general equilibrium model with financial assets is at stake. Such formulation will raise a number of new problems that will deal mostly with the determination and representation of the nonlinearities of such model. Contrarily with what we have with demand functions where there is a large exploitable body of knowledge, the current state of knowledge on the existing computable general equilibrium models is likely to be of little help given their lack of nonlinearities, almost an artifact of these models.



## 9 Concluding comments

All too often, the attitude toward the market has been and continues to be quasi-religious. For some, competitive markets represent the ultimate form of economic perfection and questioning that perfection is iconoclastic. For others, free markets are simply the worst form of economic organization in enabling the wealthy to exploit the poor. Nevertheless, the study of the properties of markets can follow the principles of science. Ulam did not have the slightest idea of the depth of the problems raised by their study. We now understand that markets, whether they deal with real goods or with financial assets, are very complex objects.

We also know that the efficiency properties of competitive markets are counterbalanced by a variety of more or less unpleasant phenomena. Discontinuities of the equilibrium selection process is one. But there is also the existence of non Pareto efficient fluctuating equilibria in economies with stationary fundamentals when some agents face restrictions in their ability to transfer wealth between time periods. Similarly, sunspot equilibria not only exist but are not Pareto efficient when some consumers are unable to insure themselves against the realization of some states of nature, even if the latter have no impact on the fundamentals of the economy. The non-discrete structure of the equilibrium set in the case of incomplete asset markets implies that the financial and real spheres are not independent. These properties make up only a sample of what can go wrong within the market system.

In another direction, A big unknown is how real world data fit with such model and, then, what can we infer from models based on these data. This would give us a significant help in the design of the ways to prevent market economies from falling into one of the undesirable but possible states they can be trapped in. This program is likely to take more time to complete than it will take for the economy to recover from the current crisis. But this is not a sufficient reason to delay work until the next crisis.

The malfunctions of the market system do not suffice to invalidate Hayek's classic argumentation about the superiority of the decentralized market system over more centralized forms of economic organizations [26].

Markets do not always perform at their best. Our understanding of why this can happen has improved a lot during the past thirty or forty years but that has not been sufficient to prevent the current crisis to reach enormous proportions. One reason is that the understanding of the properties of the market system among the general population, politicians and even economists is not sufficiently widespread. Contrarily to what many people believe or have believed with Ulam, economic theory is not trivial, which does not facilitate its diffusion and does not help governments to take the right decisions when the latter are painful in the short run.

This superiority of the market system does not make it immune to a variety of diseases that are sources of inefficiencies. These defects were not known to Hayek when he wrote his classic defense of free markets. Nevertheless, they do not change the bulk of the argument. Further research is necessary to improve our understanding of the malfunctions of the market system in order to be able to prevent them from occurring without stifling the forces of competition<sup>2</sup>.

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