A dynamic model for rumors spread with communication costs

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Abstract

In this article we start by reviewing some mathematical models for the spread of rumors. We mention some studies carried out using graphs, stochastic and probabilistic models and also some epidemic like and evolutionary models. We analyze a repeated game with the replicator dynamics for the spread of a rumor in a society conformed by an informed population about a rumor that communicates the rumor to another population. The rumor can be communicated in its original form or with the opposite content. We compare the results obtained when only social costs are involved with the case when also communication costs are present.

Introduction

The first academic studies on the relevance of the spread of rumors were carried out by Allport and Postman (1946, 1947), Caplow (1947) and Peterson and Gist (1951).

With the emergence of new technologies for the transmission of information, the spread of rumors became an important tool to influence the opinions and decisions of individuals and groups in situations of uncertainty. Studies of rumors propagation through media, Internet, Twitter and other social networks were done by DiFonzo et al. (1994), Rosnow (1988), DiFonzo and Bordia (1998, 2000), DiFonzo and Bordia (2004, 2007), Weeks and Garrett (2014) and Maor (2020).

Rumors and misinformation also played an important role during the COVID-19 pandemic, causing major challenges for the healthcare system around the world, and also fed unreliable information on etiology, prevention, vaccines effectiveness, etc (Ali (2020), Tasnim et al. (2020), Zou and Tang (2020)).

In this paper we center our attention on the studies of dynamics models for rumor spread. The articles is organized as follows: In section 2 we mention some classical studies on stochastic models of rumors diffusion; section 3 deals with rumor spread studies based on epidemic and strategy games models; in section 4 we analyze and compare the main results of recent studies applying the replicator dynamics to modeling rumors spread with and without transmission costs; section 5 includes some concluding remarks.

Mathematical stochastic models of rumors diffusion

In this section we review the results appeared in several papers that considered mathematical models of rumors diffusion.

We start pointing out the article written by Daley and Kendall (1965), where it is compared a stochastic model or the associated deterministic model for rumor spreading. Here they presented some numerical analysis based on Monte Carlo and other calculations techniques.

Pittel (1987) studied a probabilistic model of rumor spreading in which n people know a rumor and pass it to someone chosen at random. In these papers it was considered the shortest-path problem for graphs with random arc lengths presented by Frieze and Grimmett (1985). Pittel (1990) continued the studies of the Daley-Kendall model proving the conjecture that the number of eventual knowers is asymptotically normal with mean and variance linear in N.

Molchanov and Whitmeyer (2010) presented two Markov models of the spreading of rumors. They found the limiting distribution as the population becomes large for the time to spreading of the rumor to the full population. The initial conditions about the rumor knowers vary in these two models and in each case it is studied the limiting distribution.

Junior et al. (2011) considered discrete-time stochastic systems for modeling processes of rumor spreading. Mocquard et al. (2020) made a probabilistic analysis of rumor-spreading time in a continuous-time model, extending a previous works of the same authors in the discrete time case.

Several authors studied rumors spread in the context of a social network considering different types of graphs:

Nekovee et al. (2007) introduced a general stochastic model for the spread of rumors and derived meanfield equations that describe the dynamics of the model on complex social networks. They showed that in both homogeneous networks and random graphs the model exhibits a critical threshold in the rumor spreading rate below which a rumor cannot propagate in the system.

Seo et al. (2012) modeled the social network as a directed graph and studied the rumor source in order to check if this information is in fact a rumor.

Fountoulakis and Panagiotou (2013) studied a classical push model where the information randomly spreads from an initial node in a network described by a graph. They calculated the probability that the information could be spread to all nodes. Clementi et al. (2016) also studied a push model considering edge-Markovian evolving graphs. They found that the push protocol completes with high probability in optimal logarithmic time. Mahmoud (2020) introduced a probabilistic model for the spreading of fake news.

Tripathy et al. (2010) analyzed two models where anti-rumors arisefor combating the spread of rumors on a social network. It was also studied by Chen et al. (2020) that estimated the cost and efficiency of anti-rumor messages strategies.

Epidemic like and strategy game models for rumor spread

Epidemic dynamics models have been widely applied to model the dissemination of information on social media. It was done in a similar way to how the spread of epidemics is modeled.

The susceptible–infected–recovered (SIR) model is a type of compartmental models (individuals move from compartments) in epidemiology. These models have their origins in the articles of Ross (1916), Kermack and McKendrick (1927) and Kendall (1956).

In the article by Abdullah and Wu (2011) it is used a SIR model to simulate the hotspots of Twitter. Other applications of SIR models to media information dissemination were done by Cheng et al. (2013), and Fibich (2016) that used a variation of the SIR (Bass–SIR model). This research line was followed by Zhao et al. (2012), and Zhao et al. (2013) by considering a more complex SIR model for rumor spreads analysis.

Jin et al. (2013) used epidemiological models of a system of differential equations to characterize information cascades in twitter.

Zhu et al. (2016) proposed an epidemic-like model with both discrete and nonlocal delays for investigating the spatial-temporal dynamics of rumor propagation. They analyzed the corresponding characteristic equations of the model and the stability conditions of the equilibrium. They also included some numerical simulations.

In real social network, variation in the popularity of hotspots leads to the changes in the willingness of social network users to participate in hotspots. This has been studied as a strategic game.

Li et al. (2015) considered an evolutionary game for the diffusion of rumor in complex networks.

Studies on the transmission of rumors through internet and the new social media technology networks have been done by Bloch et al. (2018) by considering a model with biased and unbiased agents, playing a game with a perfect Bayesian equilibrium.

Xiao et al. (2020) studied a group behavior model for rumor and anti-rumor, by introducing an evolutionary game, to reflect the conflict and symbiotic relationship between rumor and anti-rumor.

Some authors also authors combined evolutionary game theory with epidemic model. Li et al. (2015), Xiao et al. (2019) proposed a mixed information dynamic model for social hotspot propagation based on evolutionary games combined with the traditional SIR epidemic model.

Dynamic games and the replicator dynamics applied to modeling rumors spread

We begin this section by summarizing the main results of the article by Accinelli et al. (2021), where it is used the replicator dynamics to model rumors spread.

It is considered a population in which two types of sub-populations coexist: The first subpopulations knows the content of a certain rumor (this subpopulation is called informed, I) while the other subpopulation does not know the content (it is calls uninformed, NI). In each population there are two types of individuals: those who prefer to spread the rumor with its original content (called the diffusers, D), and those who prefer to transmit the rumor with content opposite to the original (the modifiers, denoted by O). We assume that an individual from population I (a diffuser), shares the rumor with an individual of population NI chosen at random. If the uninformed individual has the same interests in spreading the original rumor, then the rumor spreads to this individual and he also becomes a diffuser. If he has an interest contrary to the original rumor, then he becomes a modifier willing to spread the rumor with content opposite the original. On the other way, if a modifier (O) meets an individual of population NI, he shares the rumor with content opposite to the original and the uninformed individual becomes a modifier if he also has an interest contrary to the original rumor, or becomes a diffuser if he is interested in spreading the original rumor.

The payoff matrix in Figure 1 was used by Accinelli et al. (2021) to describe the utilities for each population when using the different types of strategies.

I ↓ /NI →	D	O
D	a, b	−c, −d
O	−e, −f	g, h

Table 1: Payoff matrix.

This payoff matrix defines a game with two pure Nash equilibria (Nash (1950) and a mixed equilibrium. Let us denote by:

- d_I = the amount of informed spreder interested in spread the rumer in its original versionthe amounts of informed spreder.
- $o_I = 1 - d_I$ = the amount of informed spreder interested in spread the rumer modified versionthe amounts of informed spreder.
- d_{NI} = the amount of non-informed spreder interested in spread the rumer in its original versionthe amounts of non-informed spreder. .
- $o_{NI} = 1 - d_{NI}$ = the amount of non-informed spreder interested in spread the rumer modifiedthe amounts of non-informed spreder.

The distributions $(d_{I,}o_I),(d_{NI},o_{NI})$ are an strategic profile followed by the representative individuals of each population, informed and non-informed.

1. The payoffs for the informed individual arise from the following considerations:
- $a > 0$ and $g > 0$ denote the utilities of an informed individual who supports, or respectively opposes, the original content of the rumor when he meets an individual with the same preferences.
- $c > 0$ denotes the cost of spreading the rumor by an informed individual interested in its diffusion in its original form to a non informed individual interested in modify the rumor.
- $e > 0$ denotes the cost to spread the rumor by an informed individual who is opposite to the diffusion of the rumor in its original form to anon informed individual interested in spred the rumor in its original form.
2. The payoffs for the non-informed individual are:
- $b > 0$, and $h > 0$ are the utilities of non-informed individual when receive a rumor according where her preferences.
- $f > 0$ and $d > 0$ are the utilities of individuals non-informed receiving rumors opposite to her preferences.

An informed player will choose to spread the rumor in its original form if the expected value of doing so $E^{I}(D)$ is greater than that of spreading it misrepresented, which we will denote by $EI(O)$. Analogously for uninformed players. The corresponding expected values associated with each of the possible strategies of the uninformed players by $E^{NI}(D)$ and $E^{NI}(O)$.
Where:

$$E^{I}(D) = ad_{NI} - c(1 - d_{NI}), E^{I}(O) = -ed_{NI} + g(1 - d_{NI})$$
$$E^{NI}(D) = bd_{I} - f(1 - d_{I}), E^{NI}(O) = -dd_{I} + h(1 - d_{I})$$

Players will be indifferent between the strategies if the equalities are verified.

$$E^{I}(D) = E^{I}(O) \text{ and } E^{NI}(D) = E^{NI}(O) \quad (1)$$

Solving the equalities (1), it is obtained:

$$(d_I^*, o_I^*) = \left(\frac{h+f}{b+d+f+h}, \frac{b+d}{b+d+f+h}\right) \quad (2)$$

and

$$(d^*_{NI}, o^*_{NI}) = \left(\frac{g+c}{a+c+e+g}, \frac{a+e}{a+c+e+g}\right) \text{(3)}$$

1. The solutions of these equalities correspond to a strictly mixed Nash Equilibrium $((d^*_I, o^*_I), (d^*_{NI}, o^*_{NI}))$because in addition to the equalities ($E^I(d^*_I) = E^I(o^*_I)$ and $E^{NI}(d^*NI) = E^{NI}(o^* NI)$ we have that $0 < d^*_I <$ then $0 < o^*_I < 1$ and $0 < d^*_{NI} < 1$ then $0 < o_{NI}^* < 1$.The profiles (D,D) and (O,O) are Nash equilibria in pure strategies.

Let us consider the game defined by the payoff matrix (table 1) that is repeatedly played over time. It is denoted by (dI(t),oI(t)) the percentage of informed individuals for and against the content of the rumor in a given moment, and similarly for the population of the non-informed individuals (dNI(-t),oNI(t)). The replicator dynamics shows the evolution of these percentages. In this case this dynamics is given by the system of differential equations:

$$\dot{d_I} = d_I(1-d_I)\left(E^I(D) - E^I(O)\right) \text{(4)}$$

$$\dot{d}_{NI} = d_{NI}(1-d_{NI})\left(E^{NI}(D) - E^{NI}(O)\right)$$

Where the dot above the variable indicates the derivative with respect to time. Although all the variables depend on the time to simplify the notation, we do not write the temporary variable. In extensive form this system of equations can be rewritten as follows:

$$\dot{d}_I = d_I(1-d_I)(d_{NI}H + K) \text{ (5)}$$
$$\dot{d}_{NI} = d_{NI}(1-d_{NI})(d_I H_0 + K_0)$$

where:

$$H = a + c + e + g, K = -(c + g)$$
$$H^0 = b + d + f + h, K0 = -(h + f)$$

There are the following dynamical equilibria:

(1,1), (0,0), (1,0), (0,1) (6)

that corresponds to the Nash equilibria in pure strategies

$(D_I,D_{NI}), (O_I,O_{NI}), (D_I, O_{NI}), (O_I,D_{NI})$

and the dynamical equilibrium $\left(-\frac{K'}{H'}, -\frac{K}{H}\right)$. This interior steady state corresponds to the strictly mixed Nash equilibrium of the rumors'game

((d∗I,o∗I);(d∗NI,o∗NI))

where $d_I^* = -\frac{K'}{H'}, o_I^* = (1 + \frac{K'}{H'}$ and $d_{NI}^* = -\frac{K}{H}, o_{NI}^* = 1 + o_{NI}^*$. Note that, for every H,K,H⁰ and KA˘ the inequalities $0 < -\frac{K'}{H'}, \; -\frac{K}{H} < 1$ hold. So it is an interior point of the square C.

To analyze the stability of the interior equilibrium it is considered the first order approximation of this system

$$\begin{bmatrix} \dot{d}_I \\ \dot{d}_{NI} \end{bmatrix} = J(d_I^*, d_{NI}^*) \begin{bmatrix} d_I - d_I^* \\ d_{NI} - d_{NI}^* \end{bmatrix}$$

$$J(d_I^*, d_{NI}^*) = \begin{bmatrix} (1-2d_I^*)(d_{NI}^*H + K) & d_I^*(1-d_I^*)H \\ d_{NI}^*(1-d_{NI}^*)H' & (1-2d_{NI}^*)(d_I^*H' + K') \end{bmatrix} (7)$$

where is the Jacobian evaluated at each equilibrium point.
The HartmanGrobman theorem or linearization theorem is used to obtain the linear approximation to analyze the stability. The linear system corresponding to the system (5) evaluated at $\left(-\frac{K'}{H'}, -\frac{K}{H}\right)$ is $J\left(-\frac{K'}{H'}, -\frac{K}{H}\right) = \begin{bmatrix} 0 & -\frac{H'K}{H^2}(K+H) \\ -\frac{HK'}{H'^2}(K'+H') & 0 \end{bmatrix}$
The eigenvalues of this matrix are:

$$\lambda = \pm\sqrt{\frac{K'K}{HH'}(K'+H')(K+H)}. (8)$$

Since the expression in the radical is positive, we have that one eigenvalue is positive and the other negative. Then the interior equilibrium is a saddle point.

The equilibria (1,1) and (0,0) are attractors and the steady states (0,1) and (1,0) are saddle points. Figure 1 shows the phases diagram with axes (d_I,d_{NI}). These steady states correspond to the Nash equilibria (D,D), (0,0) and ((d^*_I,o^*_I),(-d^*_{NI},o^*_{NI})) respectively.

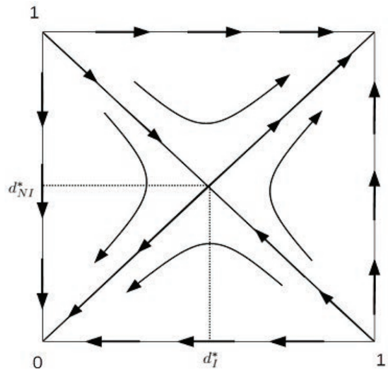
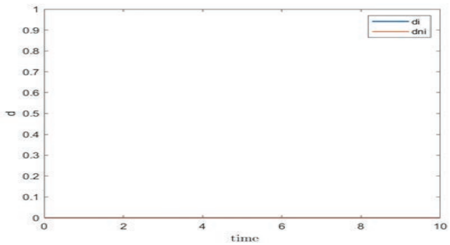


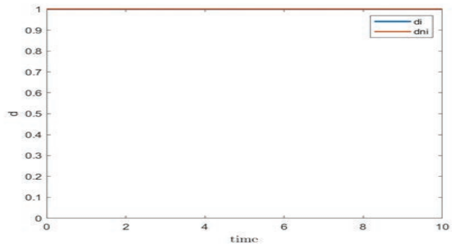
Figure 1: The phases diagram for the dynamics of the rumors.

The figure shows the evolution of the rumor spreading game. This d^*_I decreases as b + d increase and d^*_{NI} decreases as a + c increases.

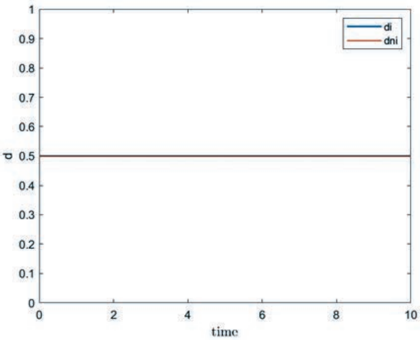
Based on the Runge-Kutta method we present some numerical solutions for the system and we plot the trajectories of the solutions for different values of the parameters presented in the model.



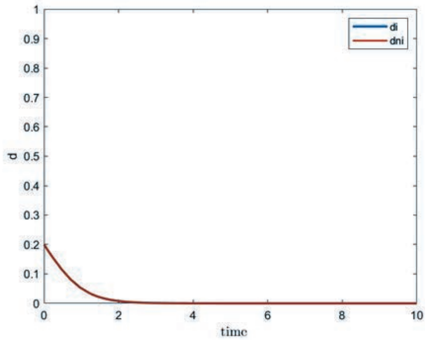
(a) $d_I(0) = 0, d_{NI}(0) = 0$



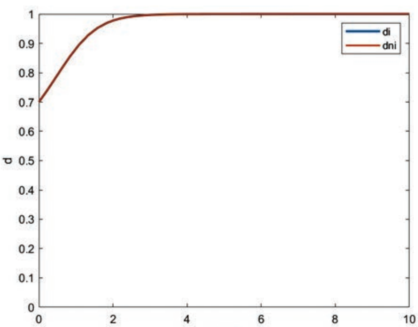
(b) $d_I(0) = 1, d_{NI}(0) = 1$



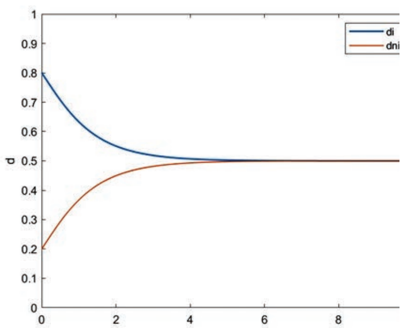
(c) $d_I(0) = 0.5, d_{NI}(0) = 0.5$



(d) $d_I(0) = 0.2, d_{NI}(0) = 0.2$



(e) $d_I(0) = 0.7, d_{NI}(0) = 0.7$



(f) $d_I(0) = 0.8, d_{NI}(0) = 0.2$

Figure 2: Trajectories of solutions according to initial conditions.

In figure 2(a) it is observed that when the utilities and the costs are the same for all the strategies, and initially the individuals of both populations are modifiers, then they mantain their strategy and the proportions remain unchanged over time. It corresponds to one of the pure strategy Nash equilibria and an it resulted an attractor. Thus no individual has incentives to modify his strategy.

In figure 2(b) it is shown the case when the costs are also equal for all the strategies but now the individuals of both populations are diffusers. This is another Nash equilibria in pure strategies and it is an attractor. Thus no individual finds incentive to distort the rumor.

In figure 2(c), when there are identical profits and costs, the proportion of diffuser individuals in each population is 0.5. Here again the agents does not modify their strategy. This corresponds to the mixed Nash equilibrium and there is no incentive to deviate from it.

In figure 2(d), the proportion of diffuser in each population is 0.2, which is less than the mixed equilibrium point. Over time, the proportion of individuals diffusers tends to 0. The modifiers, predominant in both populations, tend to “infect” the diffusers. The preferred behavior is for both populations to adopt the same strategy.

In figure 2(e), the behavior is analogous to the previous one, but in this case the diffusers ”infect” the modifiers.

In figure 2(f), 80% of informed individuals are diffusers and 20% of the uninformed individuals are diffusers. Over time it converges to the mixed equilibrium point.

We analyze how changes in profits or costs affect the trajectories of the solutions, starting from an initial distribution of 0.5 diffusers

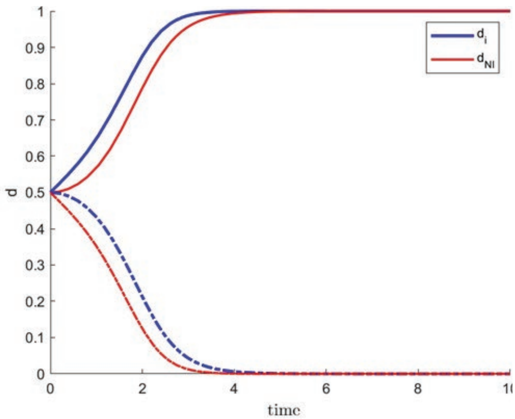


Figure 3: Trajectories when modifying profits or costs

When the utility a (solid line) is increased, it is observed that the informed population increases the preference for spreading the original rumor. Then the uninformed population is encouraged to adopt this same strategy. When the cost f (dotted line) increases, the uninformed population is discouraged from spreading the original rumor (see figure 3).

Figure 4 shows what happens when parameters a and c are modified simultaneously.

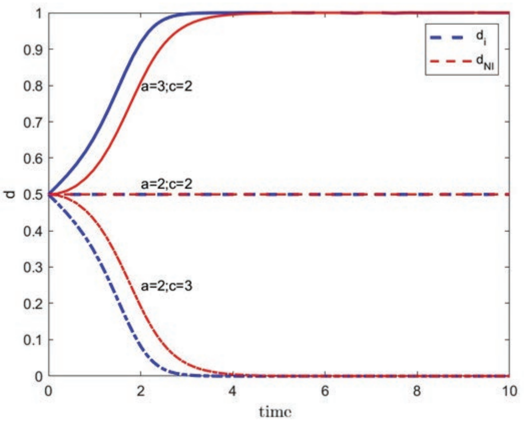


Figure 4: Trajectories when modifying relationships between utility and cost

When a and c take the same value, the populations do not deviate from their initial proportions: informed players are not incentivized nor dissuaded from their initial strategy.

When the profit a is greater than the cost c, the informed players are encouraged to adopt the strategy of spreading the original rumor so it converges to the Nash equilibrium point (1,1). Consequently, the uninformed players also prefer to adopt this strategy.

When the profit a is less than the cost c, the informed players are encouraged to adopt the strategy of distorting the rumor and it converges to the Nash equilibrium point (0,0). Consequently, the uninformed players also prefer adopt this strategy.

Here we present an extension of the model studied by Accinelli et al. (2021) by including the costs of communication faced by the players. Thus the payoff matrix resulting is that shown in Figure 5.

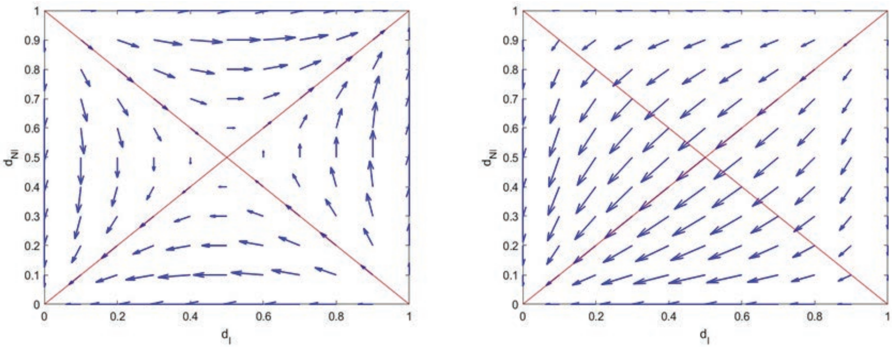
$I \downarrow / NI \rightarrow$	D	O
D	$a-p, b-p$	$-c-r, -d-r$
O	$-e-s, -f-s$	$g-q, h-q$

Table 2: Payment matrix when communication involves costs

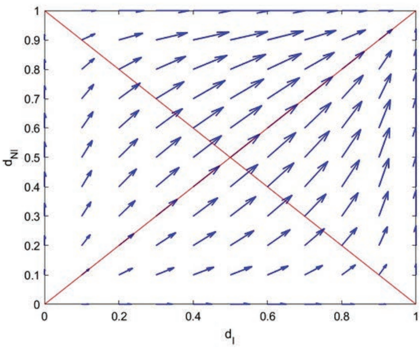
The parameters *a, b, c, d, e, f, g* and *h* are the same as defined above. Now the following costs are defined:

$p > 0$ is the cost derived from the time invested in the transmission between an informed diffuser to a
non informed diffuser. $q > 0$ is the cost derived from the time invested in the transmission between an informed modifier to *a*
non informed modifier. $r > 0$ is the cost derived from the time invested in the transmission between an informed diffuser to *a*
non informed modifier. $s > 0$ is the cost derived from the time invested in the transmission between an informed modifier to a non informed diffuser.

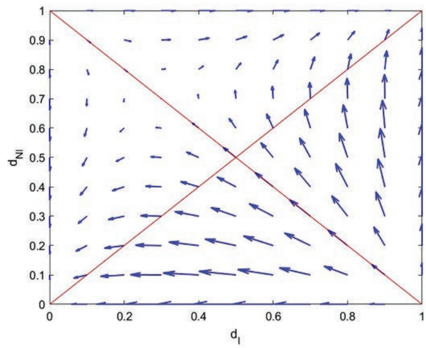
The study of the replicator dynamics can be carried out in a similar way to the previous case. Figure 5 shows the phase diagrams arising by assuming different cost *p, q, r* and *s*.



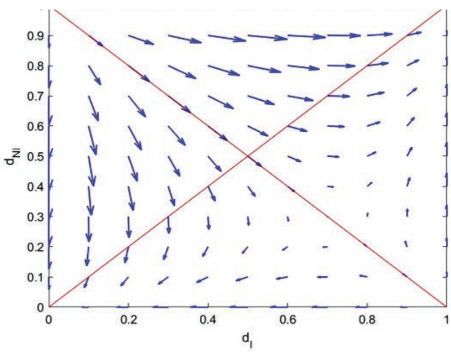
(a) $a,b > p$ and $g, h > q$, (b) $a, b < p$ and $g, h > q$



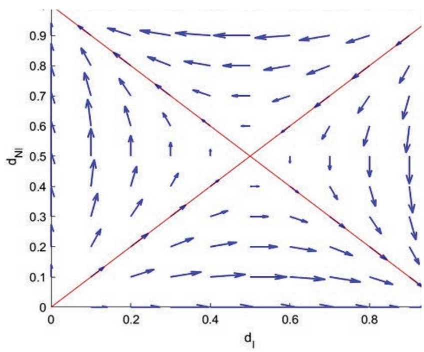
(c) *a, b* > *p* and *g, h* < *q*



(d) *a, b* > *p* and g, h > *q*. Increase *r*



(e) *a, b* > *p* and *g, h* > *q*. Increase *s*



(f) *a, b* < p and *g, h* < *q*. *r, s* fixed

Figure 5: trajectories of solutions according to initial conditions.

In figure 5(a), under the initial assumptions it is observed that the solutions approximate the points (0,0) and (1,1) where both players play the same strategy.

In figura 5(b), it is assumed that *a, b* < *p*. By increasing the costs of the strategy where both agents are diffusers, the solutions approach (0,0) where both players are modifiers.

In figure 5(c), it is assumed that *g, h* < *q*. This implies an increase in the cost of the strategy where both players are modifiers, thus the solutions tend to

(1,1) where both players prefer to be diffusers.
In figure 5(d), the cost r of the strategy is increased where the informed agents are diffusers and the uninformed ones are modifiers. This implies that players prefer using strategies where both spread the rumor with the same content ((0,0) and (1,1)).
In figure 5(e), the cost s is increased. This situation is analogous to the previous one and the players will choose to switch to strategies where both are diffusers or modifiers.
Finally, in figure 5(f), it is assumed that $a, b < p$ and $g, h < q$. This means increasing the costs when both players play the same strategy. Therefore, in the phase diagram it is observed that the solutions approach to (0,1) and (1,0), where players choose to play opposite strategies.
An important observation regarding the inclusion of communication costs arises from the differences that appear in the phase diagrams. This affects the dynamic of the game. If certain costs were high enough, strategies that in the previous model were not profitable, could be useful in the model with communication costs.

Concluding Remarks

The study of the spread of rumors and anti-rumors has become a very important issue that has been modeled with different mathematical techniques. In this paper we have reviewed several mathematical models for the spread of rumors, including stochastic, probabilistic models, SIR models and dynamic evolutionary models.

We compared two models of rumor spreading described by repeated games with the dynamics of the replicator. We contrasted the results obtained when, besides the social costs of spreading a rumor, also transmission costs were considered. In the latter case, the dynamic of the game was modified and this affected the agent's preferences regarding the optimal strategies for spreading the rumor.

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