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Two popultions, two strategies and a conflict: An evolutionary approach

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Abstract

In this paper we study the mathematical foundations of different evolutionary models that analyze the evolution of a society composed of two populations with antagonistic interests. Populations are involved in a conflict whose solution depends on the action chosen by each individual. None of them are able to impose a solution on the other population. Two individuals, one from each population, are randomly chosen to play a game. In a strategic way, each individual of each population, must choose one of two feasible strategies. The solution obtained for each population will be the result of the aggregated action of the individuals.

1 Introduction

Several topics of the social sciences and in particular of the economic theory, have been modeled considering two populations with opposing interests confronted in a conflict whose solution depends on the strategic choice of each one of the individuals that make up this population. In their simplest form each of these individuals choose between two possible strategies.

The choice results in the strategy that each individual understands as the best for their own interest in the context of the conflict in which they are immersed. As long as the conflict is repeated in period after period, they will be forced to choose a different strategy at each stage from the one previously chosen or to continue with the previous behavior. According to the strategic choice of the individuals of each population, it will be divided into two subpopulations, whose real sizes are modified according to the choice made by them. That is, each subpopulation, within each population, will be characterized by the strategy followed by those who compose it. A generalized version of this approach can be found in [Borkar V. S., Jain, S and Rangarajan, G].

There is a wide literature in which applications of the dynamics of the replicator to population genetics and in general to biology are analyzed. An interesting survey on this topic is introduced in [Hofbauer, J., and Sigmund, K.]. In this work we are interested in the applications to society, in particular, in social conflicts whose evolution can be analyzed using this dynamic and whose results can be modified from a successful choice of model parameter values.

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The evolution of a society composed by two populations facing a conflict as above characterized has been modeled using evolutionary games theory, and in particular the dynamics of the replicator. Some works following this approach are for instance, [Accinelli,E.; Covarrubias, E.], [Accinelli E. y O. Salas.], [Accinelli, E.;Sánchez Carrera, E,;Policardo, L,; Salas, O.], [Accinelli E,; Carrera, E.] and [Accinelli,E.; Martins, F.; Oviedo, J.; Pinto,A.; and Quintas, L.;]. Extension of this theory to other games are analyzed in [Cressman, R. and Tao, Y].

The rest of the paper is organized as follows. In section 2 we consider the model for a two populations game. In section we introduce the replicator dynamics. We analyze the stability of its equilibria and their relationship with the Nash equilibria. In section we give the final remarks.

2 The model

The society consists of two populations denoted by I and II. Individuals in each population can choose between two possible strategies. According with the choice made by each individual of each population, they will form a subpopulation. These two populations are immersed in a conflict that is repeated continuously and no population is able to impose a solution to the other.

The conflict develops as follows. Each individual in each population faces an opponent from the other population, both are chosen at random. This couple of individuals play a game. The individual payoffs depend on the strategy followed by each individual of this couple. The results for each population is the aggregate payoff obtained by each individual. The game is repeated continuously. At the end of each stage each player in each population can redefine their strategy. According with this election, the relative sizes of each subpopulation will be modified. The total amount of individuals in each population are assumed constant, however the percentage distribution between sub-populations may change over the time.

At each time the model can be summarized by a normal form game, represented in the following table:

	II_1	II_2
I_1	a_{11}, b_{11}	a_{12}, b_{12}
I_2	a_{21}, b_{21}	a_{22}, b_{22}

By $a_{ij},\ i,j = 1,2$ we denote the payoff for a player of the population I playing the $i-$th strategy facing and individual of population II playing the $j-$th strategy. Analogously the payoff for the individuals of population II are denoted by b_{ij}.

According to Von Neumann Morgenstern's theory, the strategic choice of each individual will be made for the expected value of each possible pure strategy to be chosen. We denote by $P_{Ij}, j \in \{1,2\}$ the probability that an individual in the population I will choose the strategy j. Similarly P_{IIj} will represent the probability that an individual in the population II will choose the strategy j. It follows that $p_{I2} = 1 - p_{I1}$ and $p_{II2} = 1 - p_{II1}$.

It is assumed that these probabilities correspond to the percentage distribution of individuals in each population following, at any given time, each of the pure strategies. That is to say that $p_{ij} = N_{IJ}/N_I$ where N_{Ij} denote the number of individuals that in the I population follow the strategy j at any given time, and N_I is the amount (constant) of individuals in the population I.

We make similar assumptions for population II. Although these values change over time, in that follows, in order to simplify the notation we omit the temporal variable.

The expected value of the strategies $i = 1, 2$ of a player from population I are respectively

$$E_{I1} = (a_{11} - a_{12})p_{II1} + a_{12}$$

$$E_{I2} = (a_{21} - a_{22})p_{II1} + a_{22}.$$

Analogously for players of the population II,

$$E_{II1} = (b_{11} - b_{12})p_{I1} + b_{12}$$

$$E_{II2} = (b_{21} - b_{22})p_{I1} + b_{22}.$$

We assume that the values of the payoffs for each population are different from each other. So, at most, there is only one mixed Nash Equilibrium strategy and it is given by the solutions of the following system equations:

$$\begin{aligned} E_{I1} &= E_{I2} \\ E_{II1} &= E_{II2} \end{aligned} \tag{1}$$

According with our model the simultaneous solution is given by

$$x^* = H/K \ \ and \ \ y^* = H'/K'.$$

Where

$$H = (a_{11} - a_{12}) - (a_{21} - a_{22}), \quad K = a_{22} - a_{12},$$

$$H' = (b_{11} - b_{12}) - (b_{21} - b_{22}), \quad K' = b_{22} - b_{12}.$$

There a mixed Nash Equilibrium strategy if and only if:

$$0 < x^* = H/K < 1 \ \ and \ \ 0 < y^* = H'/K' < 1. \tag{2}$$

If the inequalities are not verified then there is not mixed Nash Equilibrium.

3 The replicator dynamics

The concept of replicator dynamics assumes two or more large population of replicators, in which different types meet in proportion to their share in each population. This meeting - i.e. the interaction of different replicators (e.g. different strategies in a game) - generates payoffs. Replicators reproduce with regard to their payoffs in relation to the payoffs of others. The general idea is that replicators whose payoffs are larger (smaller) than the average payoffs of the population will increase (decrease) their share inside each the population. Summarizing, according with the replicator dynamics, the proportion of individuals who follow the most successful behavior tends to increase in each population. While the proportion of individuals who follow behaviors with lower than average results tends to decrease.

This evolutive process is modelled by the replicator dynamics. In the case of games involving two population whose individuals have two feasible strategies, this dynamics is given by the next system of differential equations:

$$\begin{aligned} \dot{x} &= x(1-x)[Hy-K] \\ \dot{y} &= y(1-y)[H'x-K'] \end{aligned} \tag{3}$$

The dynamical equilibria are $(0,0),(1,1),(1,0),(0,1))$ and (x^*,y^*).
The linear approximation of this system is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} (1-2\bar{x})[H\bar{y}-K] & \bar{x}(1-\bar{x})H \\ \bar{y}(1-\bar{y})H' & (1-2\bar{y})H'\bar{x} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{4}$$

The eigenvalues are given by the equation

$$det \begin{bmatrix} (1-2x^*)[Hy^*-K]-\lambda & x^*(1-x^*)H \\ y^*(1-y^*)H' & (1-2y)H'x^*-\lambda \end{bmatrix} = 0. \tag{5}$$

We have the following possibilities:

1. The simplest cases correspond to those in which there are dominant strategies. In this case the inequalities given in (2) are not verified.

    - The stationary points are given by $(0,0)$, $(0,1)$ $(1,0)$ or $(1,1)$. There exist only one Nash equilibrium, depending on the values of the parameter H, K, H' and K' which will be one of these dynamical equilibria. In addition it will be globally asymptotically stable for the replicator in the interior of the square $[0,1]\times[0,1]$.

2. The other cases are those in which the inequalities (2) are verified. So, we can write the equalities

    $$K = x^*H, \text{ and } K' = y^*H'. \tag{6}$$

    Thus, the following equations hold:

    $$K+H = (1-x^*)H, \text{ and } K'+H' = (1-y^*)H' \tag{7}$$

    where $0 < x^* < 1$, $0 < y^* < 1$. Substituting (6) and (7) in equation (5), we have that the eigenvalues are given by

    $$\lambda = \pm\sqrt{x^*y^*(1-x^*)(1-y^*)HH'}$$

    Note that the sign of the expression under the radical depends only on the sign of the product HH'. So, we have the following situations:

    (a) If $HH' > 0$ then we have two real eigenvalues and the next two possibilities:

i. As in [Accinelli, E,; Bazzano, B.; Robledo, F. and Romero, P.] the game has two complementary equilibria being one of them Pareto superior: This occurs if for instance $a_2 > c_1,\ a_1 > b_1$ and $d_1 > c_1,\ d_2 > b_2$ then: (I_1, J_1) and (I_2, J_2) are Nash equilibria. Note that under the hypothesis of this model we have that

$$H > 0,\ K < 0,\ H' > 0,\ K' < 0,\ \ H + K > 0,\ H' + K' > 0.$$

Then we have two real eigenvalues with different signals.

ii. It corresponds to a situation which there are two Nash equilibria exist in pure strategies: 1) Individuals of the population II prefer strategy 2 and the individuals in population I prefer strategy 1., or reciprocally. In general if $b_2 > a_2,\ b_1 > d_1$ and $c_2 > d_1,\ c_1 > a_1$.

$$H < 0,\ K > 0,\ H' < 0,\ K' > 0,\ \ H + K < 0,\ H' + K' < 0.$$

Even in this case the eigenvalues will have the form given by the equation (5) and from the mathematical point of view we will find that these two equilibria are both asymptotically stable for the dynamics of the replicator. Depending on the initial conditions, it will converge to one or another equilibrium.

(b) If $HH' < 0$ then there is no Nash Equilibrium in pure strategies, and the only mixed Nash Equilibrium is the center of a set of continuous periodic orbits. The position of the center will depend on the values of the parameterd.

4 Concluding Remarks

We have made an exhaustive analysis of the interaction of two populations, where individuals in each population can choose between two possible strategies. These two populations are immersed in a conflict that is repeated continuously. We considered the different cases that arise from the use of replicator dynamics. We pay special attention to the study of the stability of the equilibria corresponding to the dynamics of the replicator, and their relationship with the Nash equilibria of a conflict modeled as a normal two populations two strategy game.

This analysis could be useful for application in several cases in which two populations are involved in a conflict with similar characteristics to those studied here

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