TWO SOURCES OF LOW-DEVELOPMENT TRAPS FROM A HUMAN CAPITAL PERSPECTIVE

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Abstract

This paper analyzes two possible sources of low-development traps related to human capital accumulation. The first one comes from a mismatch between the skills workers have acquired through their formal education and the skills demanded by some non-innovative firms in the labor market. The second one comes from segmentation in the educational system, such that the children of better educated parents receive a higher quality education than the children of less educated parents. Two different models are presented in which each of these sources cause, correspondingly, dual economies and low-development traps.

JEL Clasification: O41, O1, I2

Key words: low-development traps, human capital, endogenous growth, innovation, quality of education

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1 Introduction

Since the early economic development theory (such as Rosenstein-Rodan, 1943), the idea of poverty or low-development traps has been present either implicitly or explicitly. The essential idea is that 'poverty begets poverty', i.e. there are self-reinforcing mechanisms that cause poverty to persist (Azariadis and Starchuski, 2005; Kraay and McKenzie, 2014). The notion of poverty traps has been useful to try to understand the huge disparities between countries' per capita levels, as well as the existence of polarized groups of households within countries: rich *vs.* poor. As emphasized by Kraay and Mc Kenzie (2014), the implication of the existence of poverty traps is that one-time policy efforts that break the poverty trap may have long-lasting effects.

Typically poverty trap models rely on some departure form the neoclassical assumptions, which may be scale economies, positive externalities, increasing returns in the modern sector of the economy vs. constant returns in the traditional sector, imperfect competition, borrowing constraints, nutritional poverty traps, corrupted or weak institutional frameworks and geographical poverty traps.[1] More recently, some *behavioral* poverty trap mechanisms have also been proposed. In general, these mechanisms lead to an "S-shape" relationship between income today and income in the future (Banerjee and Duflo, 2011), and therefore the emergence of

[1] Geographical poverty traps refer to households in remote and isolated (rural) areas being unable to choose a better production technology simply because it is not available. The idea is that a similar household placed in a better endowed area could make the 'jump' to a better production technology, and get out of poverty.

multiple equilibria, at high and low income levels, so that if an economy (or a household) starts below a certain threshold, it remains trapped in a ‘bad equilibrium’.

There is a vast literature on poverty traps models.[2] This includes the model by Murphy, Shleifer and Vishny (1989) and Matsuyama (1995), in which multiple equilibria are generated by complementarities in investment decisions in physical capital; Gali (1995) in turn analyses the emergence of a low development trap in a environment of imperfect competition in which the size of markups decreases with the capital stock and the marginal product of capital and the real interest rate does not decrease with the capital stock.

Noteworthy, in terms of growth determinants, Nelson and Phelps (1966) consider that it is the *stock* of human capital what leads to growth, since it affects the ability to innovate, whereas Lucas (1988) considers that it is the *accumulation* of human capital, the factor that leads to growth.[3] Naturally, poverty trap models based on human capital accumulation have also been proposed, as it is the case of Kremer (1993), Galor and Zeira (1993), Barham et al. (1995), Redding (1996), Acemoglu (1997), Galor and Tsiddon (1997) and Berti Ceroni (2001). Some of these models emphasize complementarities between different investment decisions; others emphasize the decision of human capital accumulation in a context of credit constraints. In turn, the models by Accinelli, Brida and London (2007), and Heymann, Galiani, Dabus and Thome (2006) find the growth process to require the achievement of threshold levels in human capital alongside thresholds in other economic variables. This seems to be in line with the development path followed by some Latin American countries such as Argentina. It also coincides with empirical evidence by Azariadis and Drazen (1990), who find that in a sample of 29 countries, no country with a low ratio of literacy to GDP was able to grow quickly in the period 1960-80 and Ros (2003), who shows evidence that even high initial levels of education are not a sufficient condition for achieving high growth rates.[4]

In Ikegami et al (2016)’s model individuals are endowed with different levels of innate ability and stock of capital, have to choose between two alternative technologies for generating income

[2] For a very complete review, see Azariadis and Starchuski (2005); for a more succinct yet more recent critical assessment, see Kraay and McKenzie (2014). Banerjee and Duflo (2011) provide an insightful and intuitive description of a wide range of potential poverty trap mechanisms.

[3] These were proposed as an alternative growth source to Solow (1956)’s technological change.

[4] Other related evidence can be found in Mariscal and Sokoloff (2000), and Krueger and Lindahl (2001), among others.

and face missing financial markets.[5] In this setting, low ability households remain poor because they never find the high-return technology attractive. Intermediate ability households can either end up chronically poor or may take up the high-return technology and move out of poverty, depending on their initial ability and assets endowments. Finally, individuals above both the critical level of assets and ability will always be non-poor.[6] Pugliese et al (2017) offer a novel perspective; they argue and show empirical evidence that countries that have a higher number and/or more sophisticated capabilities involved in producing the country's products, can start their industrialisation process at lower levels of GDP per capita.[7] On the contrary, countries with poorly diversified economies cannot take off until they have reached an extremely high level of capital.

Laajaj (2017) models a behavioural poverty trap in which currently poor individuals foresee an unfavourable future of poverty and this induces them to reduce their time horizon, which in turn leads to a lower accumulation of assets, reinforcing their poverty condition.[8] Dalton et al (2016) model the way in which an aspirations failure can lead to a poverty trap. A person's aspiration level can spur greater effort but it can also produce a low satisfaction from a particular outcome. If individuals take aspirations as given when choosing effort, that is, if individuals fail to internalise the feedback from effort to aspirations, they will end up in a low-wealth, low-aspirations, low-effort trap.

While there have been numerous and diverse proposed models on poverty trap mechanisms, empirical work on the prevalence of poverty traps has grown at a lower rate. This is no surprise given that an strict assessment and testing of poverty traps requires either panel data or randomized control trials followed over some reasonable period of time, and these kind of data is not abundant. In their assessment of the available empirical evidence on poverty traps, Kraay and

[5] Both technologies are capital using and skill-intensive but the high-technology is subject to a fixed cost, such that it is not worth using at low levels of capital.

[6] This model is particularly thought-provoking because, when the authors introduce the possibility of households experiencing a shock, they conclude that shocks (even ex-ante, i.e. if not materialised) have more dramatic effects on individuals of intermediate ability: the possibility of a shock can set them on a different accumulation path, moving them to the low-equilibrium track. In this context, the authors conclude (after a simulation exercise) that cash transfer programs that prioritise the vulnerable over the chronically poor may be preferable to programs focused on the chronically poor, in the sense that the first design reduces more the mid-term poverty rate (although it increases the short term one).

[7] The degree of country fitness is measured by the structure of exports in terms of their diversification and complexity of the manufacturing process.

[8] The author offers supportive empirical evidence from an agro-input subsidy program and a matched savings program implemented in Mozambique, under a randomized controlled trial. Improvements in economic prospects increased the time horizon of poor beneficiaries and their asset accumulation over the two years after the intervention.

McKenzie (2014) remain skeptical because there is no conclusive evidence for many of the common poverty-trap causing mechanisms considered in theoretical models, although evidence seems more conclusive in supporting behavioural and geographical poverty traps.[9] However, as these authors acknowledge, the mixed evidence on poverty traps does not invalidate these models. On the one hand, poverty trap mechanisms have usually been tested in isolation while in reality it is likely that more than one reinforcing mechanism is at work. Second, even if households or countries are not in a poverty trap, they may be converging to a steady state only at a very slow rate (Kraay and McKenzie, 2014, p145); thus aid programs can be justified. Surely, more work is required on this front and advances in data availability will enable that.

In this paper, two sources of poverty traps are explored, building upon previous models. Both sources are related to the accumulation of human capital. In one of the models, which builds on Redding (1996) as well as on London et al. (2008), there is a complementarity between investments in research and development by firms and investments in human capital by workers in a context of heterogeneous firms in which there are innovative and non-innovative firms. When workers are matched to non-innovative firms, they are requested to perform tasks unrelated to or too easy for their training; thus they experience a human capital loss, named here a *δ-effect*. In the other model, previously introduced by Santos (2011), there is a segmented educational system, such that the children of better educated parents receive a sensible higher quality education than the children of less educated parents, also provoking a *human capital loss* among disadvantaged children, called here a *ω-effect*. The *δ-effect* can be framed in Nelson and Phelps (1966)'s perspective, because it refers to a matching problem between the already acquired abilities and the demanded abilities by the labor market, which in turn, affects the innovation effort performed by firms. On the other hand, the *ω-effect* can be framed in Lucas (1988)'s perspective, since it addresses a problem in the process of human capital accumulation, such that it produces a high degree of heterogeneity or polarization in cognitive skills acquisition among different groups of society.

The rest of the paper is organized as follows. Section 2 presents an endogenous growth model in which the existence of innovative *and* non-innovative firms together with a potential human capital loss effect discourages human capital investment and increases the chances of a dual

[9] These mechanisms include savings-based poverty traps, increasing returns to scale in the modern sector, nutritional poverty traps, and poverty traps arising from the interaction between borrowing constraints and a non-convex production technology.

equilibrium. Section 3 presents a poverty-trap model in which the existence of another kind of human capital loss effect, related in this case to a low quality education, the *ω-effect*, leads the fraction of the population that starts with a level of human capital below a certain threshold to remain there forever, driving the economy to a lower aggregate income per capita. Finally, Section 5 highlights some links across the two models as well as with previous literature and presents the concluding remarks.

2 Simple growth model I: firms heterogeneity and the δ-effect as sources of a low-development trap

This model builds upon Redding (1996)'s model, in which an economy's deficiencies in education and training are intimately related to firms' investments in product quality, in the form of profit-seeking Research and Development (R+D hereafter). It also builds upon a previous version by London (2007) and London et al. (2008). The model consists of a continuum of non-overlapping generations of workers and entrepreneurs living for two periods, and these are matched one-to-one in both periods of life.

The main difference between Redding's model and the one presented here is as follows. Redding assumes firms to be homogeneous in the first period, that is, all use the same technology, but some invest in R+D for the second period –with some probability of success, while others do not. In this model on the contrary, firms are heterogeneous even in the first period: there are innovative and non-innovative firms, and thus they use two different technologies in the first period. The innovative firms invest in R+D for the second period, again with an associated probability of success. Workers are randomly assigned to innovative and non-innovative firms in both periods of life, and the allocation in the second time period is independent of the first one. However, the second difference is that if workers are assigned to non-innovative firms in their second period of life, they experience a human capital loss, named as a *δ-effect*, because despite having invested in human capital they end up employed in a firm that demands less qualifications and skills than the ones they owe.

2.1 The model

The model assumes a closed economy, with no government, composed of entrepreneurs (or firms) and workers, both in a continuum of non-overlapping generations; with each generation of workers and entrepreneurs living for 2 periods.

The representative worker's lifetime utility of generation t is given by:

$U_t(c_{1,t}, c_{2,t}) = c_{1,t} + \rho c_{2,t}$ (2.1)[10]

where $c_{j,t}$ is the consumption of generation t in period j, and ρ is the time discount factor, with $0 < \rho \leq 1$. It is assumed that individuals are risk neutral and that there is no human capital depreciation. As in Redding (1996)'s model, individuals inherit or are born with one unit of human capital $h_{1,t}$ (from investment in human capital of the preceding generation), and invest a fraction u of their time in period 1 to increase this human capital stock.[11] Then:

$h_{2,t} = (1 + \gamma u^{\theta})h_{1,t}$ (2.2)

where γ is a parameter of efficiency of the educational system, with $0 < \gamma$, and θ is the human capital accumulation intensity, with $0 < \theta < 1$. Parameters γ and θ represent the social and institutional conditions of the economy, correspondingly. Equation (2.2) indicates that the human capital in the second period of life of each generation depends on the fraction of time devoted to study u, the efficiency of the educational system γ, and the efficiency of the institutional arrangements θ.[12]

The economy produces one homogeneous final good. Departing from Redding's model, here there are two kinds of firms or entrepreneurs: innovative and non-innovative ones. Innovative firms invest in the first period a fraction α of their output on R+D. If this investment is successful in producing a new technology, which occurs with a probability μ (with $0 < \mu < 1$), it induces a jump in productivity. More precisely, technology A moves through a 'quality ladder', in 'steps' produced by R+D, and given by a parameter $\lambda \geq 1$. Normalising the very first period technology to $A_0 = 1$, the quality of the period 1 technology employed by innovative entrepreneurs of generation t is given by $A_{1,t} = \lambda^t$, where t denotes the number of innovations that have occurred. Thus, in this model, t denotes not only the generation number but also a technology level. In this way, in the first period ($j = 1$) of each generation t there are two levels of technology used in the economy: A_t is the technology used by innovative firms whereas non-innovative firms use an older technology A_{t-1}.[13] In the second period ($j = 2$), innovative firms that have been successful in their R+D investment, use a technology λA_t, with $\lambda \geq 1$; innovative firms that have not been successful keep using technology A_t, and non-innovative firms remain with their obsolete

[10] Following Aghion and Howitt (2001), there is perfect complementary between $c_{1,t}$ and $c_{2,t}$. This means that individuals decide over consumption independently of time and of previous decisions; i.e. intertemporal valuation is only considered through ρ.

[11] It is assumed that only high school and college education have an opportunity cost of studying given by the forgone wage in the labour market.

[12] For simplicity, it is assumed that human capital cannot be obtained through learning-by-doing, nor *via* spillover effects between firms.

[13] In the very first period of life of this economy, non-innovative firms use technology $\lambda^{-1}A_0$.

technology A_{t-1}; that is, in the second period there are three technology levels. Also note that because of the normalization used, each technology level can always be expressed in terms of A_{t-1} times the corresponding λ factor. The production technology is given by a linear function in all cases, which is described below.[14]

Each worker is randomly matched one-to-one with an entrepreneur both in period 1 and period 2 of their lives. However, in period 2, if they are matched with a non-innovative firm, they experience a skill-loss, represented by a parameter δ, with $0 < \delta \leq 1$: workers with high education are matched with firms that need less human abilities.[15] The lower is parameter δ, the higher is the skill-loss, or *δ-effect*. This effect is not present in the first period because workers only count with the inherited human capital level.

In this way, the production function of non-innovative (NI) firms is given by:

$y_{j,t}^{NI} = A_{j,t-1}h_{j,t}$ for $j = 1$ (2.3a)

$y_{j,t}^{NI} = A_{j,t-1}\delta h_{j,t}$ for $j = 2$ (2.3b)

Note the 'skill-loss' effect in the second period. In turn, the production function of innovative (I) firms is given by

$y_{1,t}^{I} = A_{j,t}h_{j,t}$ for $j = 1$ (2.4a)

and, in the second period:

$y_{1,t}^{I} = \lambda A_{j,t}h_{j,t}$ for $j = 2$ with probability μ (2.4b)

$y_{1,t}^{I} = A_{j,t}h_{j,t}$ for $j = 2$ with probability $(1-\mu)$ (2.4c)

From previous equations and assuming there are a non-innovative firms and $(1-a)$ innovative ones, the overall production level in each period is given by:

$y_{1,t} = [a + (1-a)\lambda]A_{j,t-1}h_{1,t}$ (2.5a)

$y_{2,t} = [a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)]A_{j,t-1}h_{2,t}$ (2.5b)

The level of production is higher the higher the proportion of innovative firms in each period. Additionally, in the second period, the level of production is higher the lower the δ-effect (with no human capital loss occurring when $\delta = 1$), and the higher the probability of investments in R+D being successful.

The worker's problem

[14] This is in line with an AK model (Aghion and Howitt, 2001)

[15] The model does not consider unskilled work.

Individual workers can obtain wage in period 1 which is a fraction β of the technology per unit of human capital (the other $(1-\beta)$ is obtained by the firm). This wage will depend on whether they are employed by an innovative or a non-innovative firm, and this is a random process with probability given by the proportion of each kind of firms in the economy.

The a fraction of workers who are employed in a non-innovative firm, have a wage given by:

$w_{1,t}^{NI} = \beta A_{t-1}$ (2.6a)

whereas the $(1-a)$ fraction of workers who are employed in a innovative firm, have wage given by

$w_{1,t}^{I} = \beta A_t = \beta\lambda A_{t-1}$ (2.6a)

This wage is the opportunity cost of studying, and clearly it is higher for those workers who are lucky to be employed in an innovative firm. In period 2, workers are again matched with either an innovative firm or a non-innovative one, and it is assumed that this allocation is independent of their previous employment.[16] Following equations (2.6), the expected wage can be obtained as a time discounted β-fraction of the possible technologies considering the probability μ of innovative firms being a successful in their innovations, as well as the probability a of the worker being matched to a non-innovative firm (*vs.* a probability being $(1-a)$ of being matched with an innovative firm).

$E[w_{2,t}] = \rho\beta A_{t-1}[a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)]$ (2.7)[17]

The intertemporal budget constraint is different for workers that are initially employed in a non-innovative firm from those who are initially employed in an innovative firm. Each of these budget constraints is given by:

$c_{1,t} + \rho c_{2,t} \leq w_{1,t}^{NI}(1-u)h_{1,t} + \rho E[w_2]h_{2,t}$ (2.8a)

$c_{1,t} + \rho c_{2,t} \leq w_{1,t}^{I}(1-u)h_{1,t} + \rho E[w_2]h_{2,t}$ (2.8b)

Putting things together, individuals will make their decisions about the amount of time u devoted to study maximizing the expected discounted lifetime income, which can be obtained replacing (2.2) and the corresponding versions of equations (2.6) and equation (2.7), on the right hand side of the corresponding expression (2.8), depending on whether they are employed by a non-innovative or an innovative firm in their first period of life:

[16] *A priori*, this seems a strong assumption, but it may not be so if –as it evidenced later- one considers that workers employed in innovative firms in the first period have lower incentives to invest time in education for their second period of life.

[17] Note that, for convenience, the equation is expressed in terms of A_{t-1} technology, considering that innovative firms employ $A_t = \lambda A_{t-1}$ technology if they are unsuccessful, and $\lambda A_t = \lambda^2 A_{t-1}$ technology if they are successful.

For workers who are employed by a non-innovative firm in their first period of life, their optimization problem is given by:

$$max_u^{NI} \quad \beta A_{t-1} h_{1,t}[(1-u) + \rho(1+\gamma u^\theta)[a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)]] \quad (2.9a)$$

For workers who are employed by an innovative firm in their first period of life, their optimization problem is given by:

$$max_u^{NI} \quad \beta A_{t-1} h_{1,t}[\lambda(1-u) + \rho(1+\gamma u^\theta)[a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)]] \quad (2.9b)$$

The first order condition of both versions of equation (2.9) yields the optimal fraction of time in the first period devoted to human capital accumulation. For workers who are employed by a non-innovative firm in their first period of life, this fraction is given by

$$u_{NI} = \left[\left(a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)\right)\rho\gamma\theta\right]^{\frac{1}{1-\theta}} \quad (2.10a)$$

In turn, for workers who are employed by an innovative firm in their first period of life, this fraction is given by

$$u_I = \left[\frac{\left(a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)\right)\rho\gamma\theta}{\lambda}\right]^{\frac{1}{1-\theta}} \quad (2.10b)$$

Given that u^{NI} and u^I are the proportion of time devoted to study by workers, both expression (2.10a) and (2.10b) must be bounded between 0 and 1, otherwise it would be unfeasible. It can be seen that for any given set of parameter values, $u_I < u_{NI}$ exactly in $(1/\lambda)^{\frac{1}{1-\theta}}$; that is, workers who are employed in the innovative firms in the first period of time will invest a lower fraction of their time in studying, which is intuitive, as they have a higher opportunity cost of studying in the first period given by their higher wage. The higher the technological jump of each innovation (λ), the bigger wage difference between innovative and non-innovative firms, and thus, the lower the time allocated to study by workers employed in innovative firms as compared with workers employed in non-innovative ones.

It is also worth noting that –in general– both equation (2.10a) and (2.10b) show that the time devoted to study is an increasing function of the probability of innovating successfully μ, the proportion of innovative firms in the economy, and the efficiency of the educational system γ. Additionally, u is higher the higher the value of parameter δ, that is, the lower is the human capital loss if the worker ends up employed in an non-innovative firm in the second period of his life.

The firm's problem

As stated above, there are innovative and non-innovative firms or entrepreneurs and it has been assumed that there are a non-innovative firms, and $(1-a)$ innovative firms. But what makes a firm become innovative?; i.e. what can make fraction $(1-a)$ increase? Their decision is based on comparing the benefit of being innovative with the benefit of being non-innovative, and this is also influenced by the amount of time their workers spent studying. Innovative firms have a benefit given by the following expression, in which firms consider the time allocated to work by their employed workers $(1-u_I)$ in the first period and –given that employment in the second period of life is independent of employment in the first period– it incorporates the *expected* human capital investment of all workers for the second period:

$$\pi^I = (1-\beta)\left\{\left[(1-\alpha)\lambda(1-u_I) + \rho(\mu\lambda^2 + (1-\mu)\lambda)\left(a\left(1+\gamma u_{NI}^{\theta}\right) + (1-a)\left(1+\gamma u_I^{\theta}\right)\right)\right] A_{t-1}h_{t,1}\right\} \quad (2.11a)$$

In turn, non-innovative firms stay producing with the old technology. As before, their benefit considers the time allocated to work by their employed workers $(1-u_{NI})$ in the first period, and incorporates the *expected* human capital investment of all workers for the second period:

$$\pi^{NI} = (1-\beta)\left\{\left[(1-u_{NI}) + \rho\left(a\left(1+\gamma u_{NI}^{\theta}\right) + (1-a)\left(1+\gamma u_I^{\theta}\right)\right)\right] A_{t-1}h_{t,1}\right\} \quad (2.11b)$$

As in Redding's model, the decision to be an innovative firm or not depends on whether $\pi^I - \pi^{NI} > 0$. Such condition holds when the following condition is satisfied:

$$(1-\alpha)\lambda(1-u_I) - (1-u_{NI}) + \rho\left[\left((\mu\lambda^2 + (1-\mu)\lambda) - 1\right)\left(a\left(1+\gamma u_{NI}^{\theta}\right) + (1-a)\left(1+\gamma u_I^{\theta}\right)\right)\right] > 0$$

This can be re-expressed as:

$$(u_{NI} - \lambda u_I) + (\lambda - 1) + \rho\left[(\lambda-1)(\lambda\mu+1)\left(a\left(1+\gamma u_{NI}^{\theta}\right) + (1-a)\left(1+\gamma u_I^{\theta}\right)\right)\right] > \alpha\lambda(1-u_I) \quad (2.12)$$

This is a more complex condition than the one obtained by Redding's. On the right hand side of the inequality there is the firm's 'cost' of investment in R+D times the time devoted to work in period 1 by workers, which is part of the worker's opportunity cost of studying, augmented by the higher technological step λ these firms use. For firms to become innovative and be willing to invest in R+D, this cost needs to be exceeded by the left hand side of the expression. The first term of the left hand side is the gap between time devoted to study by workers employed in non-innovative firms and time devoted to study by workers employed in innovative firms. Although

$u_I < u_{NI}$, given that u_I is augmented by the higher technological step λ innovative firms use, this term is likely to be negative. The key term is thus the discounted value (ρ) of the probability of success in R&D (μ) times the technological 'jump' ($\lambda - 1$), times the expected increase in human capital from period 1 to period 2, which is analogous to Redding's condition.

In general, and coinciding with Redding's condition for investment in innovation, a favorable decision to innovate is more likely to occur the less the future is discounted (i.e. the higher is ρ), the higher the probability of success in investing in R+D (μ), the higher the size of quality jumps in the technology (λ), the higher the education productivity parameter γ, and the higher the elasticity of human capital increase to time spent in education θ.

As in Reddings's model, it is also worth noting that there is a clear interdependence between the firms' decision to invest in R+D and the workers' decision to invest in their human capital accumulation. In fact, these decisions are taken simultaneously and thus each part's decision (firms and workers) will be taken depending on what they expect the other part to do. In this model, workers' decision are influenced by more factors: their initial employment (differing from Redding's), their expectation on whether they will be by an innovative firm in their second period of life and if so, whether the firm will be successful in its innovation. In turn, firms consider how much time workers will decide to devote to study to increase their human capital and to which worker they will be matched.

Because of this strategic complementarity and simultaneity of decisions, both "good" and "bad" equilibriums may be generated simultaneously. In the "good" equilibrium condition (2.12) is satisfied: workers expect firms to successfully invest in R+D and to be employed in one of them and thus they invest more time studying, and firms expect to benefit from workers' higher human capital, and thus they both materialise these investments. In the "bad" equilibrium the condition (2.12) is not satisfied: workers do not expect firms to invest, or they do not expect them to be successful, or they do not expect to be lucky to be employed in one of the innovative firms in their second period of life and thus allocate small amounts of time to increase their human capital. Moreover, workers have a lower incentive to invest in studying when they are employed in innovative firms in their first period, somehow creating a paradox, as this choice of time allocation reduces the incentives of firms to become innovative. In turn, when firms do not expect workers to invest sufficiently in human capital, they do not find incentives to invest in R+D.[18] It is also worth noting that while in (2.12) the δ-effect does not play a direct role, it plays an indirect role via the decision of workers on the time spent studying, which is analysed below.

2.1 The influence of the δ-effect on the possible equilibriums

As stated above, the condition for a "good equilibrium" to arise is given by expression (2.12). Also, a "good" and a "bad" equilibrium may arise, depending on the expectations of entrepreneurs and workers about each other, as well as on the different parameters of the economy. Also, the worker's decision is given by expressions (2.10a) or (2.10b), depending on whether the worker has been employed in a non-innovative or an innovative firm in the first period. Considering these expressions one can evaluate the influence that introducing firms heterogeneity and the associated δ-effect has on the decision to invest in human capital and thus on the satisfaction of the condition for firms to become innovative, expression (2.12). Table 1 summarises the four possible cases under different parameter values of the economy that workers may expect.

Table 1: Optimal amount of time devoted to human capital accumulation under different parameter values

	All innovative firms and absence of human capital loss $\boldsymbol{\delta = 1, a = 0}$	**Some non-innovative firms (heterogeneous firms) and presence of human capital loss** $\boldsymbol{\delta < 1, a > 0}$
No successful innovations $\boldsymbol{\mu = 0}$	$u_R = [\rho\gamma\theta]^{\frac{1}{1-\theta}}$ (2.13)	$u_{NI} = [(a\delta + (1-a)\lambda)\rho\gamma\theta]^{\frac{1}{1-\theta}}$ (2.14a) $u_I = \left[\left(a\frac{\delta}{\lambda} + (1-a)\right)\rho\gamma\theta\right]^{\frac{1}{1-\theta}}$ (2.14b)
Some successful innovations $\boldsymbol{\mu > 0}$	$u_R' = \left[\left((\mu\lambda + (1-\mu)\right)\rho\gamma\theta\right]^{\frac{1}{1-\theta}}$ (2.15)	$u_{NI} = \left[\left(a\delta + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)\right)\rho\gamma\theta\right]^{\frac{1}{1-\theta}}$ (2.16a) $u_I = \left[\left(a\delta/\lambda + (1-a)(\mu\lambda^2 + (1-\mu)\lambda)\right)\rho\gamma\theta\right]^{\frac{1}{1-\theta}}$ (2.16b)

Suppose workers expect that there will be no successful innovations, that is, $\mu = 0$ (first row of the table). Within this pessimistic case, one may think of two possibilities: 1) all firms are

[18] Redding (1996) offers a detailed description of the possible equilibria configurations. This is left for future work the analogous analysis.

innovative ($a = 0$) and thus there is no δ-*effect*, 2) firms are heterogeneous ($a > 0$) and thus there is a δ-*effect*. In the first case, when all firms are innovative, despite not being successful in their innovations, there is no effective human capital loss, and the time spent studying is given by equation (2.13) in Table 1. This coincides with the amount of time devoted to study by workers when firms do not innovate in Redding's model (that's the reason for subindex R). In the second case, when some firms are non-innovative, there are two expressions for the time devoted to study, one for workers who are employed by non-innovative firms in the first period (u_{NI}), and another for workers who are employed by innovative firms in the first period (u_I). In the case of the u_{NI}, given by expression (2.14a), the difference with equation (2.13) is given by the term: $(a\delta + (1 - a)\lambda)$. Given that $\lambda > 1$, and the other parameters of this term are lower than 1, it can be verified that $u_{NI} > u_R$. That is, the time devoted to study by workers that are employed in non-innovative firms in the first period is higher when firms are heterogeneous than when firms are all innovative, despite the fact that workers factor in the potential skill-loss they may experience (the δ-*effect)* if they are employed in a non-innovative firm in their second period. The intuition is that their opportunity cost for investing in human capital is lower than when all firms are innovative, and the potential payoff for becoming employed in an innovative firm in the second period is attractive enough to invest (even when they consider innovations not to be successful), somehow compensating the potential δ-*effect*. However, the opposite occurs for workers who are employed in innovative firms in their first period of life; it can be verified that $u_I < u_R$. For them, the opportunity cost of investing in human capital is higher, they expect no successful innovations, and factoring in the potential δ-*effect* reduces their incentive to devote time to study. The ultimate aggregate value of u will depend on the proportion of each type of firm a and $(1 - a)$. And thus a paradox emerges: the higher the proportion of non-innovative firms a, the higher the proportion of workers who will invest more in their human capital, and thus the more likely is that the condition for a firm to become innovative (condition 2.12) holds. Yet as the proportion of innovative firms increases, more workers will tend to invest less in human capital –as they are paid higher salaries in their first period anyway– and firms will have lower incentives to become innovative. In this way, firms heterogeneity seems to reinforce heterogeneity. This result is in line with Aghion and Howitt (2001): the strategic complementarity between the workers' choice of education and the firms' R&D decisions may lead to a low-development trap.

Now suppose that workers expect some innovations to be successful, that is when $\mu > 0$. Again, there are two possibilities, depending on whether there are heterogeneous firms, and thus a δ-

effect) or not. When all firms are innovative, the time devoted to study is given by (2.15) in Table 1, the u'_R case, which coincides with Redding's fraction of time devoted to study when firms innovate. In turn, when firms are heterogeneous, as before, there are two expressions for the time devoted to study: one for workers who are employed by non-innovative firms in the first period (u_{NI}) (expression 2.16a), and another for workers who are employed by innovative firms in the first period (u_I) (expression 2.16b). For an analogous reasoning to the one above, one can verify that $u_{NI} > u'_R > u_I$. In words, in presence of heterogeneous firms, workers employed in non-innovative firms in the first period of life will invest more in human capital than when all firms are innovative, yet, in the same heterogeneous context, workers employed in innovative firms in the first period will invest less amount of time in studying than when all firms are homogeneous and innovative. As before, the opportunity cost of studying for workers employed in innovative firms in their first period of life is higher, plus there is the potential δ-*effect* if they end up employed in a non-innovative firm in their second period of life. As in the case of no successful innovations, the ultimate aggregate value of u will depend on the proportion each type of firm a and $(1 - a)$, and –again– a higher proportion of innovative firms actually discourages becoming innovative.

Comparing this with Redding's results, it seems that considering firms heterogeneity in the first period, and the δ-*effect* reduces the likelihood of occurrence of a good equilibrium. Moreover and paradoxically, the higher the proportion of innovative firms in the first period of life of each generation, has an adverse effect on increasing such proportion. Looking in a different way at these results, in an economy with marked differences among firms (innovative and non-innovative), if workers ignore the δ-*effect* assuming or believing that all firms are the same, they will all devote the same amount of time to studying u_R or u'_R and this will lead more likely to a good equilibrium with firms becoming innovative. But with successful innovations, workers who are unfortunately matched with non-innovative firms in the second period of life will notice the wage differential in the second period and this will lead future generations to factor-in the δ-*effect* in their expected salary equation, which in turn will lead future generations to allocate differential amounts of time to study depending on their first work experience leading to a more uncertain equilibrium.

Relatedly, empirical evidence suggests that in Latin America secondary school drop out is still a significant problem, given by the high opportunity cost and the low probabilities of getting an employment in the more dynamic industries (Carlson 2002). Additionally, UNESCO (2013) states that while secondary education has expanded in Latin America between 2003 and 2013 (reaching 77% in 2011), it has done so at a lower speed than over the previous decade. The same report indicates that countries that implemented a cash transfer program conditioned on attending school were able to increase enrolment much faster than countries which did not introduced such kind of programs, supporting the relevance of the opportunity cost in the decision-making of time devoted to study. This report also highlights that it is 'transferable skills' such as problem solving, the ones that are vital for adapting to different working contexts, and these are acquired mostly at upper secondary level, in which the region is still far from universal enrolment.

3 Simple growth model II: the ω-effect as a source of a low-development trap

This model is based on Berti Ceroni (2001) and has been proposed by Santos (2011). It is a poverty trap model with two sources: the initial unequal distribution of human capital and income, and the differences in the quality of education received by the children of parents with a higher educational level and the children of parents with a lower educational level. This difference in the quality of education makes that given two children with the same level of investment in years of education, they may end up with very different cognitive skills. That is why this called a *potential human capital loss*, or, for brevity, the *ω-effect*. The difference with Berti Ceroni (2001) in that here the quality of education is introduced as a key determinant of the poverty trap. While Berti Ceroni arrives to a two simultaneous and stable equilibriums, one *good* and one *bad*; here there are three simultaneous and stable equilibriums, two *bad* and one *good*.

As in the case of Berti Ceroni (2001), but differing form the previous model, this is a model of overlapping generations. Each family is composed of two individuals, father and son. Each individual is born with the same ability, lives two periods and is endowed with one unit of time in each period. Individuals can make decisions only in the second period of their lives. When young, individuals can get education if their parents decide to do so. In that case, they assign their unit of time to school. Departing form the previous model, in this case, the unit of time assigned to education is indivisible. Children that do not go to school acquire a fixed level of human capital as a consequence of the passage of time. In the second period of their lives all individuals offer their time unit in the labor market, earn an income that is proportional to their level of human capital and decide how to allocate this income between consumption and spending in their children's education so as to maximize utility.

The utility function of parent *i* in time t depends on consumption in period t and on the stock of human capital of the *i*th child in period $t+1$. It takes the form:

$U^i(c_t^i, h_{t+1}^i) = \ln(c_t^i) + \tau h_{t+1}^i$ (3.1)

where τ is a parameter that measures the altruistic motive, with $0 \le \tau \le 1$. This utility function is the same than the one used by Berti Ceroni. The human capital production function presents the first departure from Berti Ceroni's model. It is assumed that there is segmentation in the economy between families with more educated parents and families with less educated parents. This segmentation is a usually observed characteristic, especially in developing countries and it can be seen in terms of social circles or networks (rich people are usually linked to other rich people while poor people have friends and family that are usually also poor), and even neighborhoods. Superscript *j* denotes the social circle.

$$h_{t+1}^{ij} = \begin{cases} \eta^j & e_t^{ij} \le b^j \\ \ln[q^j(e_t - b^j) + v^j] + s^j & e_t^{ij} > b^j \end{cases} \quad (3.2)$$

with $\eta^j = \ln(v^j) + s^j$.

h_{t+1}^{ij} is the level of human capital that the son of father *i*, from social circle *j* will acquire. η^j is the level of human capital that the child gets if he does not receive any formal education. This level varies with the social sector to which the father belongs to. It is assumed that, given two fathers, one with a higher educational level than the other, $h_t^{i1} < h_t^{i2}$, if none of the fathers decides to provide formal education to their children, the son of the better educated father (h_t^{i2}) will enjoy a level of human capital equal or greater than the level of human capital of the son of the less educated father (h_t^{i1}), that is: $\eta^1 \le \eta^2$. Parameter η^j depends on v^j and s^j, where v^j represents the knowledge and basic abilities provided at home, and it is assumed that for $h_t^{i1} < h_t^{i2}$, $v^1 \le v^2$. In turn, s^j represents the socio-economic environment in which the family lives. It is plausible to assume that when children grow up in better educated social networks, they enjoy positive externalities. The exchange with educated adults and children whose parents have high education reinforces the knowledge and skills they learn both at school and at home. Formally, this parameter moves upwards the whole human capital production function. As before, it is assumed that for $h_t^{i1} < h_t^{i2}$, $s^1 \le s^2$.

Education is public; this is a second difference with Berti Ceroni (2001)'s model. However, there exists a private cost of education e_t^{ij}, given by the cost of complementary goods such as books and transportation to school, and by the opportunity cost of non-working. This cost is assumed to be independent of the father's level of education. However, b^j is a parameter which depends on the social circle to which the child belongs to. It is the threshold of spending in education that is necessary to do such that the child's human capital starts to increase. In other words, it is the spending level at which spending in education starts to be *effective*. As before, for $h_t^{i1} < h_t^{i2}$, $b^1 \leq b^2$. This implies that the minimum level of education (and so the minimum educational spending) that the children of better educated parents require so that an improvement in their education is observed, is higher than the minimum required by the children of less educated parents.

Finally, q^j represents the quality of education that the child belonging to social sector *j* receives. This parameter is of particular interest in the model, since it constitutes one of the causes of low-development traps. The quality of education that the child receives is not a decision variable for the father. It is determined by the allocation of public resources to each school. It is assumed that for $h_t^{i1} < h_t^{i2}$, $q^1 \leq q^2$. This implies that schools with students coming from better educated families have better quality of education than schools with students coming from less educated families.

As in Berti Ceroni (2001)'s model, the economy produces one only final good through a linear technology that uses human capital as the only production factor:

$$Y_t = H_t = \int_I h_t^i g_t(h_t^i) dh_t^i \quad (3.3)$$

where H_t is the aggregate stock of human capital in period *t* and $g_t(h_t^i)$ is the density function that characterizes the distribution of human capital among fathers in period *t*, such that $g_t(h_t^i) \geq 0$ y $\int_I g_t(h_t^i) dh_t^i = 1$. The distribution of human capital in the initial generation of fathers is exogenously given: $g_0(h_0^i)$, with $h_0^i \in (\sigma, \varepsilon)$ y $\eta_1 = \sigma < \varepsilon$.

The individual maximization program that father *i* has to solve in time *t* is given by:

$$\max_{e_t^{ij}} \quad U^{ij}(c_t^{ij}, h_{t+1}^{ij}) = \ln(c_t^{ij}) + \tau h_{t+1}^{ij}$$

s.t (4.4)

$$c_t^{ij} = h_t^{ij} - e_t^{ij}$$

$$h_{t+1}^{ij} = \begin{cases} \ln(v^j) + s^j & e_t^{ij} \le b^j \\ \ln[q^j(e_t^{ij} - b^j) + v^j] + s^j & e_t^{ij} > b^j \end{cases}$$

$$(e_t^{ij}, c_t^{ij}) \ge (0,0)$$

Following Behrman and Birdsall (1983), it is assumed that school quality can not be influenced by parents' spending in their children's education. Therefore the budget constraint does not consider taxes. Replacing the budget constraint and the human capital production function in the utility function and maximizing with respect to e_t^{ij}, the expression of optimal spending in education is given by:

$$e_t^{*ij}(h_t^{ij}) = \begin{cases} b^j & h_t^{ij} \le \bar{h}^j \\ \dfrac{\tau h_t^{ij} + b^j}{(1+\tau)} - \dfrac{v^j}{q^j(1+\tau)} & h_t^{ij} > \bar{h}^j \end{cases} \quad (3.5)$$

where $\bar{h}^j = b^j + \dfrac{v^j}{\tau q^j}$ (3.6)

Note that, for human capital levels equal to or lower than the threshold $\bar{h}^j$, the education spending function is constant at b^j, the minimum required level of spending such that the human capital of children starts to increase. For human capital levels over the threshold $\bar{h}^j$, the proportion of income assigned to education is increasing in the educational level of the father. This is because the utility function is non-homothetic. This is analogous to Berti Ceroni (2001). As expression (3.6) shows, the father's human capital threshold level $\bar{h}_j$, at which education spending starts to be increasing, is increasing in b^j and in v^j, and decreasing in the quality of education q^j and parents' altruism τ.

Replacing (3.5) in (3.2), the transition equation that describes the evolution of dinasty *i*'s human capital can be obtained:

$$h_{t+1}^{ij} = \phi^j(h_t^{ij}) = \begin{cases} \eta^j & h_t^{ij} \le \bar{h}^j \\ \ln\left[\dfrac{q^j \tau(h_t^{ij} - b^j) + \tau v^j}{(1+\tau)}\right] + s^j & h_t^{ij} > \bar{h}^j \end{cases} \quad (3.7)$$

Under the mentioned assumptions regarding the parameters, the dynamics of human capital accumulation of each dynasty is independent of the aggregate dynamic, but it is dependent of the social circle *j* to which the dynasty belongs. This transition function $\phi^j(h_t^{ij})$ has a positive slope and is concave for $h_t^{ij} > \bar{h}_j$.

Following Berti Ceroni (2001), it is assumed that current income distribution determines the future one:

$$g_t(h_t^{ij}) = g_{t+1}[\phi^{-1}(h_t^{ij})] \qquad h_t^{ij} \in [\sigma, \varepsilon] \quad (3.8)$$

3.1 Multiple equilibra and poverty traps

Assume there are only two social circles or sectors, with initial educational levels clearly differenced: $j = 1, 2$, with $h_t^{i1} < h_t^{i2}$.

Santos (2007) specifies a set of plausible conditions so that the aggregate transition function exhibits multiple equilibra. These conditions can be summarized in the following four ones:

C1) $(v^j, b^j, s^j, q^j) \ge (0,0,0,0)$.

C2) $v^1 \le v^2; b^1 \le b^2; s^1 \le s^2; q^1 \le q^2$.

C3) $\dfrac{(1+\tau)}{\tau} v^j < q^j < \dfrac{v^j}{\tau[\ln(v^j) - b^j + s^j]} \quad \forall j = 1, 2$

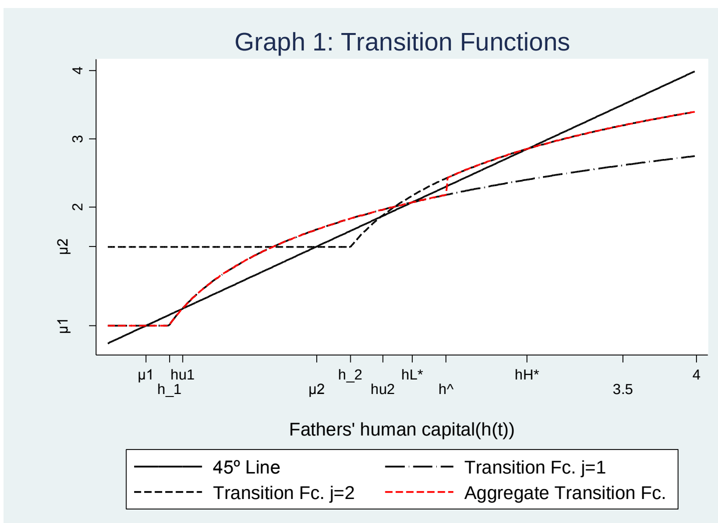
C4) $0 < (1+\tau)[\ln(v^j) - b^j + s^j] < 1 \quad \forall j = 1, 2$

Graph 1 presents one possible set of values of the parameters satisfying the mentioned conditions and producing multiple equilibra for each *j*-transition function.[19] Each individual transition

[19] The parameters' values used in the graph are:
$b^1 = 0.2; b^2 = 1.4; s^1 = 0; s^2 = 0.5; v^1 = 1.3; v^2 = 2.5; q^1 = 12; q^2 = 20; \tau = 0.5$.

function $\phi^1(h_t^{i1})$ (Transition Fc. j=1) and $\phi^2(h_t^{i2})$ (Transition Fc. j=2), shows three steady states, at human capital levels η^1, h_u^1, h_L^* y η^2, h_u^2, h_H^* correspondingly. The equilibra at human capital levels h_u^1 and h_u^2 are unstable. The other two equilibra of each *j*-curve are stable.

While each of the two *j*-transition function presents three equilibra, the equilibra that will prevail at the aggregate level will depend on the interval of human capital levels in which the $j=1$ human capital accumulation function ($h_{(t+1)}^{i1}$) operates, and the interval in which the the $j=2$ human capital accumulation function operates ($h_{(t+1)}^{i2}$). In other words, the number and type of equilibra that are determined in the economy depend on the human capital level that distinguishes between the two social circles, the more and the less educated. This threshold is called $\hat{h}$, and it can be thought as corresponding to the university education level.



Two additional conditions are necessary to allow the configuration of equilibra that interest to analyze in this paper.[20] In the first place, it is assumed that:

C.5) $\eta^2 < h_L^*$

This condition requires that the human capital level with which the children of better educated parents end up if they do not receive formal education ($e_t^{i2} < b^2$) is lower than the human capital level that the children of less educated parents get if they receive formal education. Secondly, it is required that:

[20] A detailed description of all the possible equilibra configurations is provided in Santos (2011).

C.6) $h_L^* < \hat{h}$

Together with the previous conditions, this condition guarantees the existence of three stable equilibra at the aggregate level. This can be observed in Graph 1. The red curve is the aggregate transition function: for fathers' human capital levels lower than the threshold ($h_t < \hat{h}$), the human capital accumulation function that prevails is the one corresponding to $j = 1$; for fathers' human capital levels at or above the threshold ($h_t \geq \hat{h}$), the human capital accumulation function that prevails is the one corresponding to $j = 2$. The expression for this aggregate function is given by:

$$\phi(h_t^i) = \begin{cases} \ln v^1 + s^1 & h_t^i \leq \bar{h}^1 \\ \ln\left[\dfrac{q^1\tau(h_t^i - b^1) + \tau v^1}{1+\tau}\right] + s^1 & \bar{h}^1 < h_t^i < \hat{h} \\ \ln\left[\dfrac{q^2\tau(h_t^i - b^2) + \tau v^2}{1+\tau}\right] + s^2 & h_t^i \geq \hat{h} \end{cases} \quad (3.15)$$

It can be seen that the aggregate transition function has a discontinuity at the threshold level $\hat{h}$. It can also be seen that this aggregate transition function defines three stable equilibriums at η^1, h_L^*, h_H^*, and an unstable one at h_u^1.

Dynasties with an initial human capital below h_u^1 tend, in the long run, to a steady state level of human capital given by η^1, staying forever below the level h_u^1. It is possible that the fathers of those dynasties initially invest to educate their children (with an spending level higher than the required threshold, $e_t^{i1} > b^1$), but eventually they will stop doing so because the human capital stock decreases from one generation to the other. This result is completely analogous to the first equilibrium in Berti Ceroni (2001)'s model. Dynasties whose initial human capital is above h_u^1 but below $\hat{h}$, tend in the long run to a steady state human capital level equal to h_L^*. The fathers of those dynasties invest enough in their children education ($e_t^{i1} > b^1$) and eventually the dynasty converges to the mentioned steady state.[21] Finally, dynasties with an initial human capital above $\hat{h}$, converge in the long run, to the steady state human capital level h_H^*.

In this way, in the long run, dynasties are concentrated in three groups (and not in two as in Berti Ceroni (2001)'s model): (1) *the very poor*, who in the long run do not invest in human capital above the required threshold and stay poor and without education; (2) *the poor*, who are able to invest in education and reach an income and human capital level higher than the *very poor*. However, given that they move in a low-education social circle and that they receive low-quality education, the human capital level to which they eventually converge is considerably lower than that reached by the *non poor*. This is the *ω-effect*, because if these people had had high quality education they would have reached the highest steady state equilibrium, with the highest human capital level; (3) *the non-poor*, who start with high human capital levels and their dynasties converge to a high steady state level of income and human capital, in part because of their favorable initial conditions, but also because they receive high quality education.

The first equilibrium constitutes a clear poverty trap. The second one also represents a poverty trap because although the level of human capital and income is higher than in the first equilibrium, the dynasties that reach the second equilibrium will never be able to reach the level h_H^*, only accessed by those who start with favorable conditions, belonging to the highly educated social circle.

3.2 Aggregate output in the equilibrium

From the three long run equilibra to which different fractions of the society converge, it is possible to obtain the aggregate long run output.

As in Berti Ceroni (2001), at any point in time, income distribution determines current aggregate investment in education and aggregate human capital and income of the next period.

[21]It is worth noting that there are dynasties that start with a human capital level than h_L^* (but lower than $\hat{h}$) and still they end up in the steady state human capital level h_L^*, lower than the initial level.

Considering the human capital (or income) distribution function $g_t(h_t^i)$, $G_t(\bar{h}^1) = \int_{\eta^1}^{\bar{h}^1} g(h_t^i)dh_t$, $G_t(\hat{h}) = \int_{\eta^1}^{\hat{h}} g(h_t^i)dh_t$, and $G_t(h_u^1) = \int_{\eta^1}^{h_u^1} g(h_t^i)dh_t$. Considering expression (3.5), the threshold's localization ($h_L^* < \hat{h}$), and the $\int_I g_t(h_t^i)dh_t^i = 1$, the long run aggregate output level is given by:

$$\begin{aligned} Y_\infty &= \eta^1 G_0(h_u^1) + h_L^*[G_0(\hat{h}) - G_0(h_u^1)] + h_H^*[1 - G_0(\hat{h})] = \\ &= h_H^* - (h_H^* - h_L^*)G_0(\hat{h}) - (h_L^* - \eta^1)G_0(h_u^1) \end{aligned} \tag{3.18}$$

In this expression it can be seen that the highest potential output level h_H^* is reduced because fraction $[G_0(\hat{h}) - G_0(h_u^1)]$ of the population can only reach output level h_L^*, and fraction $G_0(h_u^1)$ can only reach output level h_L^*. This is due to the initial unequal income distribution such that not all dynasties have an initial income above $\hat{h}$, but it is also due to the existence of poverty traps at η^1 and h_L^*. Poverty traps arise because the economy is segmented between social circles and the educational system far from reducing that segmentation, it contributes to reinforce it providing better quality of education to children coming from disadvantaged households.

4 Concluding remarks

In this paper two growth models have been proposed, each with an alternative source of low-development traps. Ultimately, the paper aims to provide an explanation of why investment in human capital is sometimes not a sufficient condition for fast economic growth. In one case, this is due to the existence of an initial heterogeneity across firms –innovative vs. non-innovative – which, if internalised by workers via a potential skill-loss or *δ-effect* (essentially a mismatch between the demand and supply of skills), can lead to invest less in human capital accumulation, which in turn affects the incentives of firms to become innovative. One perhaps extreme result of this model is that a higher-but-not-total proportion of innovative firms does not lead the economy to a virtuous circle. Workers employed in innovative firms in their first period of life face a high opportunity cost of investing in human capital for their second period, and when firms factor this in, they have lower incentives to become innovative. In other words, there is some extreme form of complementarity: unless *all* firms are innovative, or at least, workers behave *as if* all firms are innovative ignoring the *δ-effect*, the economy does not take off. Many parameters interplay in this model and are conceptually connected to other proposed models. For example, the future discount factor plays a role in firms becoming innovative, which is analogous to Laajaj (2017)'s argument on a shorter time horizon creating a behavioral poverty trap. Similarly, workers expectations about future earnings can be connected to Dalton et al (2016)'s aspirations failure.

In the second model, the low-development trap source constitutes a *potential human capital loss* that individuals experiment, named in the paper as the *ω-effect*, caused by a difference in the quality of education received by children from more and less educated social circles. This difference in the quality of education makes the children from dynasties with low initial income and human capital remain there forever, or alternatively, reach an intermediate income and human capital level. However, they will never be able to reach the high income and human capital level to which the children from initially advantaged dynasties converge. This model resembles in a way the three tier initial human capital model of Ikegami et al (2016).

One may ask if the two effects can reinforce one another. That is, according to the second model, someone from a dynasty with a low initial income and human capital level would get low-quality education. Then, according to the first model, it would be possible that this individual gets a job that requires less than the (low) cognitive skills he/she has acquired. If the individual considers that possibility, he/she will under-invest in education, reinforcing the aggregate low-development trap described by any of the two models.

In terms of policy implications the models suggest, connected to the first model, that different strategies should be designed so that the labor market can signal the educational system the type of skills that are being demanded and educational programs should be revised so as to satisfy the labor demand. Additionally, cash transfer programs should be directed to poor households to reduce the opportunity cost of investing in human capital, even alongside youth employment programs that successfully allocate workers to innovative firms. In terms of the second model, educational policy should sort out the way to allocate resources among schools in such a way that education can effectively contribute to the reduction of social disparities through the provision of a high quality education, and this must be addressed at the early childhood (UNESCO, 2013). In fact, Goal 3 of the "Education for All" (EFA) UNESCO Project consists of "ensuring that learning needs of all young people and adults are met through equitable access to appropiate learning and life skills programmes" (UNESCO, 2013).

Clearly, the models can be further extended separately as well as combined. On the one hand, there is scope for further variants and more analysis on the *δ-effect* model. On the other, a model that combines the *δ-effect* with the *ω-effect* could be developed. Moreover, either separately or combined, these models would require some empirical testing for their validation. These are research lines are left for future work.

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