When to Do the Hard Stuff?: Dispositions, Motivation and the Choice of Difficulties*

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Abstract

This paper analyzes individual dynamic decisions on when to face difficult tasks. We argue how individual dispositions, that is, the expression of non-cognitive dimensions, as perseverance or confidence might drive these decisions. Specifically, when experiencing low dispositions, the decision maker gets trapped into facing easy tasks that offer low economic outcomes while when experiencing high dispositions, she is willing to always deal difficult tasks that are, in contrast, more rewarding. When outcome achievements motivate the decision maker, she decides to move from low value easy tasks to high value difficult tasks at some point.

Keywords: individual dispositions, task difficulty
JEL classification: D83, D84

Resumen

Este trabajo analiza decisiones individuales sobre el momento óptimo en el que enfrentarse a tareas difíciles. En este trabajo argumentamos cómo las habilidades no cognitivas, pueden determinar estas decisiones. Específicamente, un individuo con bajas habilidades, prefiere desarrollar tareas fáciles aunque éstas ofrezcan pagos bajos. Sin embargo, un individuo con altas habilidades prefiere desarrollar tareas difíciles que ofrecen pagos altos. Cuando la consecución de pagos motiva al individuo, entonces éste decide pasar de tareas fáciles a difíciles.

Palabras Clave: habilidades individuales, dificultad de las tareas
Clasificación JEL : D83, D84

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1 Introduction

One of the reasons as to why individuals tend to avoid difficult tasks is because they do not feel able enough to confront them. Not coping with them might, however, imply foregoing the opportunity of getting better economic outcomes, not available otherwise. As (Liebow, 1967) documents in his study of the Negro male community of Washington inner city:

"Convinced of their inadequacies, not only do they not seek out those few better-paying jobs which test their resources, but actively avoid them, gravitating in a mass to the menial, routine jobs which offer no challenge -and therefore posse not threat- to the already diminished images they have of themselves(...). Thus, the man's low self-esteem generates a fear of being tested and prevents him from accepting a job with responsibilities or, once on a job, form staying with it if responsibilities are thrust on him, even if wages are commensurably higher."

The story above offers two interesting insights. The first one is that individual dispositions might dramatically influence decisions of huge economic relevance. In this spirit, when documenting the relationship between individuals' dispositions to strive for success, and upward mobility patterns in the United States, (Atkinson and Feather, 1966) highlight how, despite of the fact that education is the main determinant of upward mobility, individual dispositions should not be neglected. In fact, 65% of the people who exhibited upward mobility patterns at a higher extent, only had high school education or less.[1] The second one is the trade-off between tasks' difficulty and their associated outcomes. While a routine job is probably more easily developed than a very demanding one, good economic outcomes, as higher wages or promotion opportunities, might only be available in the latter.[2]

Our purpose in this paper is to understand and highlight the role played by individual dispositions in shaping avoidance behavior. We interpret individual dispositions as an expression of non-cognitive abilities.[3] Examples of non-cognitive abilities are emotional stability, that manifests, among others aspects, in self-confidence and self-esteem, or conscientiousness, that manifests, among others aspects, in perseverance.[4] In order to do it, we develop a tractable model in which the decision maker, henceforth DM, who is characterized by a disposition level, decides the optimal time to deal with difficulties.

[1] To establish comparisons between the prestige of occupations of parents and sons, private households populated by people older than 21 were interviewed. Specific measures of dispositions to strive for success were collected.

[2] Also, as (Atkinson and Feather, 1966) suggest, high prestige occupations are perceived as being more difficult to attain than low prestige occupations. This hierarchy can be seen as a series of tasks in which the outcome value comes together with difficulty.

[3] We will interchangeably use the term state, disposition or simply ability when referring to the non-cognitive ability level that the individual enjoys.

[4] See (John and Srivastava, 1999) for the Big Five domains of non-cognitive abilities, their traits and facets.

Our approach is as follows. We consider a dynamic framework in which at every point in time the DM might experience two states, namely, the full capacity state and the deteriorated capacity state, with some (constant) probability. In the full capacity state she enjoys high dispositions while in the deteriorated capacity state her dispositions are low. Tasks are of two types, namely, easy and difficult. On the one hand, getting good economic outcomes is less likely under difficult than under easy tasks but on the other hand, outcomes associated to difficult tasks are more valuable than the ones associated to easy tasks. We consider that states and economic performance are positively related, specifically, the higher the DM's disposition the higher the probability of being successful when developing a task, either easy or difficult. It is worth mentioning that no effort decision is analyzed here. The only decision of the DM has to make is when to confront difficulties. We assume that once she decides to confront them, she sticks at this decision forever.

We also study the case in which the DM's disposition is sensitive to outcome achievements. As (Mruk, 2006) points out, the demands of life are not constant, so self-esteem levels will fluctuate depending on what is happening in a persons life. Redundancy, bereavement, illness, studying, gaining a qualification, parenthood, poverty, being a victim of crime, divorce, promotion at work will all have an impact on our self-esteem levels. Self-esteem levels go up and down and can change over time. Also, as (Bénabou and Tirole, 2002) point out, motivation helps individuals to persevere in the presence of setbacks. We formalize this idea by allowing the probabilities of experiencing the full and the deteriorated capacity state to evolve over time. Specifically, we assume that their value at a given period depends on their value and on the likelihood of good economic outcomes in the previous period.[5]

Our results are as follows. We find that a low disposition DM will avoid difficulties forever while a high disposition DM will cope with them since the beginning. Thus, individuals with poor abilities get trapped into low value easy tasks. However, when motivation plays a role, the achievement of good economic outcomes out of easy tasks leaves the DM with the disposition of coping with difficulties from some point in time on.[6] In line with this finding it is worth mentioning the results of a program carried out in West Bengal, by the indian microfinance institution *Bandhan*, consisting on providing extremely poor individuals with productive assets. The authors observed how people ended up working 28% more hours, mostly on activities not related to the assets they were given and that their mental health had improved. The program was considered to have injected a dose of motivation, that pushed people to start new economic activities.[7]

Our proposal is closely related to the branch of literature that links poverty and

[5]In particular we assume that probabilities evolve according to a Markov process.

[6]That is consistent with (Ali, 2011), a model in which a long-run self, the planner, has to decide, at every point in time, whether to allow the short-run self, the doer, to face a menu in which a tempting alternative is available. The planner does so whenever the doer experiences high self-control.

[7]The full article is available at http://www.economist.com/node/21554506.

psychology. For instance, (Dalton et al., 2014) discuss the importance of aspirations failure in the perpetuation of poverty. This paper, as ours, highlights the role of internal constraints as a source of behaviors that might preclude individuals from getting high welfare achievements. Their research question is, however, different from ours, whereas they focus in one particular bias, namely, aspiration failure, in a static context, we analyze the role of non-cognitive abilities on dynamic decisions. We also find relations with the literature of addiction and self-control. Specifically, (Bernheim and Rangel, 2004) study patterns of addictive behavior of a DM that operates in two modes, namely, cold and hot. When in the cold mode, the DM selects her most preferred alternative whereas when in the hot mode, choices and preferences may diverge because the DM losses cognitive control. This paper, in contrast with ours, presents a theory of addiction. Finally, (Ozdenoren et al., 2012) exhaustively account for the dynamics of self-control performance of a DM that has to choose her optimal consumption path. We depart from their approach as long as we focus on outcome achievement and motivation, and not on capacity exhaustion, as the main driver of decisions.

The paper is organized as follows. Section 2 presents the baseline model, where the probability of experiencing the full capacity state is time independent. In section the probability of experiencing the full capacity state is sensitive to outcome achievements. The dynamics of its evolution is therefore outlined. Section 4 concludes. Section 5 contains the technical proofs.

2 A model on avoidance behavior

Let s_1 and s_2 denote the two states that the DM might experience. When experiencing s_2 the DM is in the full capacity state and enjoys high abilities. When experiencing s_1 the DM is in the deteriorated capacity state, meaning that she executes her abilities poorly. At every point in time, $t \in \mathbb{Z}_+$, she has a probability $q \in [0, 1]$ of experiencing s_2. Thus, she experiences s_1 with probability $1 - q$. Tasks are of two types, easy or the difficulty level d_1, and difficult or the difficulty level d_2.

The likelihood of getting good economic outcomes is denoted p_{ij}, with $i, j = 1, 2$. Subscript i refers to the DM's state, that is, either s_1 or s_2, whereas subscript j refers to the difficulty of the task, that is, either d_1 or d_2. Probabilities are as follows: first, fixing difficulty, the likelihood of good economic outcomes increases with the DM's state. There is, in fact, a large amount of literature, see (Heckman et al., 2006) and (Balart and Cabrales, 2014), posing non-cognitive abilities as one of the factors determining performance and outcomes, for instance, in education and the labor market. Second, fixing the DM's state, the likelihood of good economic outcomes decreases with task's difficulty. The following table presents these probabilities:

Table 1. Success probabilities

$$
\begin{array}{c|ccc}
 & s_1 & & s_2 \\
\hline
d_1 & p_{11} & < & p_{21} \\
 & \vee & & \vee \\
d_2 & p_{12} & < & p_{22}
\end{array}
$$

Notice that no direct relation is established between p_{11} and p_{22}. Finally, good economic outcomes are worth just 1 unit when they are the result of developing easy tasks and $K > 1$ units when they come out of developing difficult tasks.

We make an assumption regarding the success probabilities. It captures the idea that individuals with low dispositions are more vulnerable than individuals with high dispositions to the characteristics of the tasks they deal with. For high disposition individuals, task's difficulty is less relevant than for low disposition individuals in determining their chances of success. On the domain of cognitive abilities (Gonzalez, 2005) provides experimental evidence on how increasing task difficulty, was more detrimental for low ability individuals. We formally express it as:

Assumption 1: $p_{11} - p_{12} > p_{21} - p_{22}$.

The second assumption is related to the strategies among which the DM chooses:

Assumption 2: once the DM decides to face difficulties, she commits to this decision forever.

In fact, there exists evidence showing that in many situations individuals become locked into (possibly) costly courses of action and a cycle of escalation of commitment arises. The justification of previous decisions, the necessity to comply with norms or a desire for decision consistency in the decision making process, might encourage commitment.[8] The strategies available to the DM therefore comprise choosing the point in time in which to face difficulties. We denote by (∞) the *always avoiding difficulties* strategy and by (0) the *facing difficulties since the beginning* strategy. A strategy consisting on facing difficulties from a point in time $0 < t < \infty$ on, is denoted (t).[9]

The DM behaves as an expected utility maximizer. Thus, she determines her optimal path of action at the initial point in time, taking into account her disposition, that is, the point-wise probability q of being in the full capacity state. We consider that the DM is risk neutral. We then focus on the role of dispositions without dealing with risk aversion issues.[10] Thus, the current expected utility of

[8]See (Staw, 1981) for the concept of escalation of commitment. See also (Arkes and Blumer, 1985) and (Thaler, 1980) for a justification of this phenomenon based on the sunk cost effect.

[9]It is worth to stress how the explicit introduction of time does not aim to describe the evolution of abilities along the life cycle. Time only aim to capture the point-wise choice of task difficulty.

[10]See (Tanaka et al., 2010) for a study on the relationship between poverty and risk and time preferences.

choosing an easy task at time t is $qp_{21} + (1-q)p_{11}$ and that of choosing a difficult one is $K(qp_{22} + (1-q)p_{12})$. Furthermore, let $\delta \in (0,1)$ denote the discount factor of the stream of pay-offs. We formally state the DM's problem as follows:

When experiencing the full capacity state, s_2, with probability q, the DM decides, at $t = 0$, the point in time t to face difficult tasks, in order to maximize her long-run expected utility. Specifically, she solves:

$$\underset{t}{Max}\ u((t)) = \underset{t}{Max} \sum_{i=0}^{t-1} \delta^i (qp_{21} + (1-q)p_{11}) + K \sum_{i=t}^{\infty} \delta^i (qp_{22} + (1-q)p_{12}).$$

As stated, the only decision the DM has to make is when to jump into difficulties. The second part of the sum above reflects the fact that once she decides to do so, she sticks at this decision forever.

2.1 Results

It seems intuitive that individuals who enjoy better dispositions tend to perform tasks better. In fact, it is common that people tend to avoid difficulties when they do not feel prepared to face them. The results in this section capture this idea. When the DM experiences the full capacity state with high enough probability, she will opt for difficulties since the beginning. In contrast, when the probability of experiencing the full capacity state is low enough, she will prefer to avoid them forever.

We get the results by building a function λ that depends on the success probabilities and on outcomes. It defines a domination threshold between the strategy of facing difficulties since the beginning, that is (0), and the strategy of postponing them for one period, that is (1).[11] For values of q higher or equal than this threshold, (0) is preferred to (1) and for values of q smaller than it, (1) is preferred to (0). This information will be enough to identify the optimal strategy.

Before stating the result it is worth highlighting that whenever outcomes out of difficult tasks do (respectively do not) compensate the decrease in probability of successfully dealing with them, that is, whenever $p_{11}/p_{12} \leq K$ (respectively $K \leq p_{21}/p_{22}$), the DM finds optimal to always face (respectively to always avoid) difficulties, even if she is of extreme low disposition, that is, if $q = 0$ (respectively, even if she is of extreme high disposition, that is, if $q = 1$). We then focus on the interesting case in which $p_{21}/p_{22} < K < p_{11}/p_{12}$. Results are as follows:

Theorem 1. *The DM's optimal strategy is to face difficulties since the beginning whenever she enjoys the full capacity state with high enough probability (that is,*

[11] Specifically, it is the result of equating the long-run expected utility of (0) and one of (1), under all possible q. It gives the us the q such that the DM will be indifferent between not postponing difficulties and doing so for one period. Its value is $\lambda = \dfrac{Kp_{12} - p_{11}}{(p_{21} - p_{11}) - K(p_{22} - p_{12})}$. See the proof of Theorem 1.

whenever $\lambda \leq q$) and to always avoid them whenever she experiences the full capacity state with low enough probability (that is, whenever $q < \lambda$).

Optimal paths of action are extreme behaviors. Facing difficulties from an intermediate point in time is never optimal. Notice that, as going back from difficult to easy tasks is never considered by the DM, assumption 2 is immaterial here. Notice also that if for the DM never (respectively always) postponing difficulties is optimal, this is also be the case for any DM with a higher (respectively lower) disposition. The always avoiding difficulties strategy is interpreted as procrastination on onerous tasks.[12]

The following example aims to clarify the elements of the model and the result:

Example. Consider a DM who is deciding which type of job to look for or accept. An easy (routine) job gives the DM a payoff (wage) of 1 whereas a difficult (high responsibility) job has payoff $K = 1.3$. Success probabilities in either job are:

	s_1	s_2
d_1	0.7	0.8
d_2	0.5	0.7

In this case $\lambda = 0.31$. Thus if the DM is of low enough disposition (that is, if $q < 0.31$), she finds optimal to always postpone the acceptance of the high responsibility job, whereas if she is of high enough disposition (that is, if $q \geq 0.31$), she will find optimal to deal with the high responsibility job since the beginning. We depict below the ranking of long-run expected utilities under the strategies the DM chooses among. The left figure illustrates the case in which always avoiding difficulties is optimal whereas the right figure illustrates the case in which facing them since the beginning is optimal:

Figure 1. The optimal strategy is (∞)

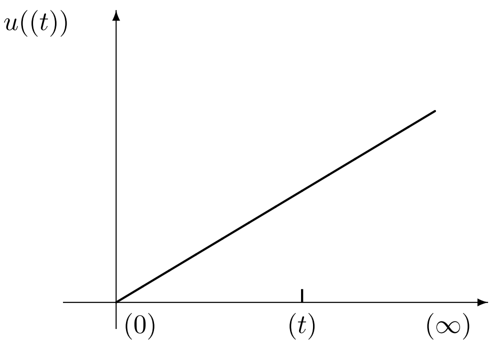
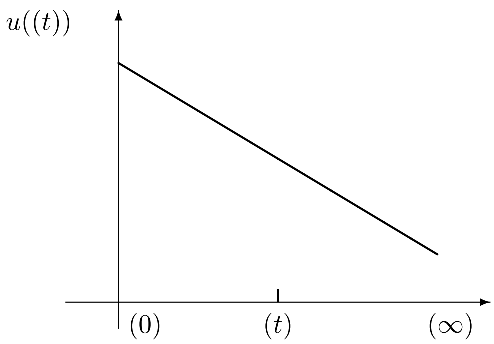


Figure 2. The optimal strategy is (0)



[12]See(O'Donoghue and Rabin, 2001) and(O'Donoghue and Rabin, 2008) for two references on procrastination.

The following remark discusses how the threshold λ reacts to the primitives of the model:

Remark. *The threshold λ is decreasing in p_{22} and p_{12} and K and increasing in p_{21} and p_{11}.*

Higher chances of successfully developing either easy or difficult tasks make the DM more prone to choose each of them. In the same vein, an increase in the value of good economic outcomes out of difficult tasks incentivizes the DM to face them. An increase in K directly raises the utility of facing difficult tasks. This happens despite of the fact that the probability of achieving good economic outcomes in these circumstances is systematically lower. The above result together and the remark are summarized as follows:

Corollary. *The range of probabilities $[q, 1]$ such that the DM's optimal strategy is to face difficulties since the beginning increases (respectively decreases) with the probability of success under difficult (respectively easy) tasks. It also increases with the value of economic outcomes out of difficult tasks.*

In what follow we present the utility associated to the aforementioned optimal strategies. We also analyze the effect of a marginal boost in dispositions, by assuming that a marginal increase in dispositions, does not affect the originally optimal strategy. For this purpose let us denote $\alpha \equiv \dfrac{1}{1-\delta}$. Results are as follows:

Proposition 1. *The long-run expected utility of any optimal strategy is monotonically increasing and linear in q. Its value is $\alpha K(qp_{22} + (1-q)p_{12})$ whenever the DM finds optimal to face difficulties since the beginning and $\alpha(qp_{21} + (1-q)p_{11})$ whenever the DM finds optimal to always avoid difficulties. Moreover, the marginal return of an increase in the DM's disposition is higher when the DM is already of high dispositions (that is, when $q \geq \lambda$) than when she is of low dispositions (that is, whenever $q < \lambda$).*

We would like to finally discuss on the possibility of carrying out a welfare assessment analysis. The intuition is as follows: consider two individuals. One of them, the disadvantaged individual, has low abilities and always avoids difficulties, the other, the advantaged individual, has high abilities and always faces difficulties. It turns out that the marginal return of boosting abilities ishigher for the advantaged individual than for the disadvantaged individual. Suppose that a social planner has one unit of resources, devoted to improve abilities. If it is the case that the planner only cares about maximizing total returns, he might allocate this unit on the advantaged individual. If he also has equity concerns, he will have to take into account that the utility gap between the advantaged and the disadvantaged individual will exacerbate. In this case, the planner might be willing to allocate resources on the disadvantaged individual.

3 The role of motivation

In this section we model how successes and failures might affect the manifestation of non-cognitive abilities. In doing so, we assume that the probability of experiencing the full capacity state varies over time according to a Markovian process. This modeling aims to capture the idea that success may boost the manifestation of the non-cognitive abilities while failure may deteriorate it. Formally, the probability of experiencing the full capacity state at time t, depends on its value at time $t-1$, and also on the success probabilities. Let $q^{(t)} \in [0,1]$ denote the probability of experiencing the full capacity state.Thus, $1-q^{(t)}$ denotes the probability of experiencing the deteriorated capacity state at time t. Let the success probabilities be the ones described in table 1 in the previous section. The following expression accounts for the evolution of the probabilities of experiencing either state:

$$[q^{(t-1)} \quad 1-q^{(t-1)}] \begin{bmatrix} p_{2j} & 1-p_{2j} \\ p_{1j} & 1-p_{1j} \end{bmatrix} = [q^{(t)} \quad 1-q^{(t)}], \tag{1}$$

where $j=\{1,2\}$ accounts for task's difficulty.[13] Consider that at time $t-1$ the DM experiences s_2 with probability $q^{(t-1)}$. Then, at time t she will experience s_2 with the probability with which she was successful in the previous period. This is captured by the first column in the matrix above. Similarly, at t she will experience s_1 with the probability with which she failed in the previous period. This is captured by the second column the matrix above. Let $q^{(0)}$ be the DM's initial probability of experiencing the full capacity state. The current expected utility of developing an easy task at time t is $q^{(t)}p_{21} + (1-q^{(t)})p_{11}$ and the one developing a difficult task is $K(q^{(t)}p_{22} + (1-q^{(t)})p_{12})$.

Notice that, as $q^{(t)}$ evolves according to a Markovian process, we can identify two stationary probabilities. These are, the one related to always facing easy tasks, denoted q^e, and the one related to always facing difficult tasks, denoted q^d. We interpret them as the average long-run frequencies with which the DM experiences the full capacity state when she always faces easy or difficult tasks, respectively. Since the likelihood of success is higher in easy tasks, it is the case that the DM is eventually better off in terms of capacities when she decides to only face easy tasks than when she decides to only face difficult tasks, more formally, $q^e > q^d$.[14] The DM's problem is as follows:

[13] Notice that $q^{(t)}$ depends on the chosen strategy. If the DM decides to face difficult tasks from $t=5$ on, $q^{(4)}$ is the resulting probability of having faced easy tasks for four periods. If she decides to face difficult tasks from $t=3$ on, $q^{(4)}$ is the resulting probability of having faced easy tasks for two periods and difficult ones from the third one on.

[14] Let $\mathbb{T}^k$, with $k=e,d$, denote the transition matrices out of facing either easy or difficult tasks, involved in expression (1), respectively. Their determinants are $T^e = p_{21}-p_{11}$ and $T^d = p_{22}-p_{12}$, respectively. In getting q^e and q^d we solve $[q^k, 1-q^k]\mathbb{T}^k = [q^k, 1-q^k]$. We have that $q^e = (p_{11})(1-T^e)^{-1}$ and $q^d = (p_{12})(1-T^d)^{-1}$. Suppose that $q^e < q^d$. This implies that $p_{11}(1-T^d) < p_{12}(1-T^e)$ or $p_{11}(1-p_{22}) < p_{12}(1-p_{21})$ which cannot hold since $p_{11} > p_{12}$ and $(1-p_{22}) > (1-p_{21})$. Thus $q^e > q^d$ has to hold.

When experiencing the full capacity state, s_2, with probability $q^{(0)}$, she decides, at $t = 0$, the point in time t, to face difficult tasks, in order to maximize her long-run expected utility. Specifically, she solves:

$$\underset{t}{Max}\ u((t)) = \underset{t}{Max} \sum_{i=0}^{t-1} \delta^i (q^{(i)} p_{21} + (1 - q^{(i)}) p_{11}) + K \sum_{i=t}^{\infty} \delta^i (q^{(i)} p_{22} + (1 - q^{(i)}) p_{12}).$$

As stated, the only decision the DM has to make is when to jump into difficulties, in an environment in which her current performance is sensitive to previous outcome achievements. As before, the second part of the sum above reflects the fact that once she decides to do so, she sticks at this decision forever.

3.1 Results

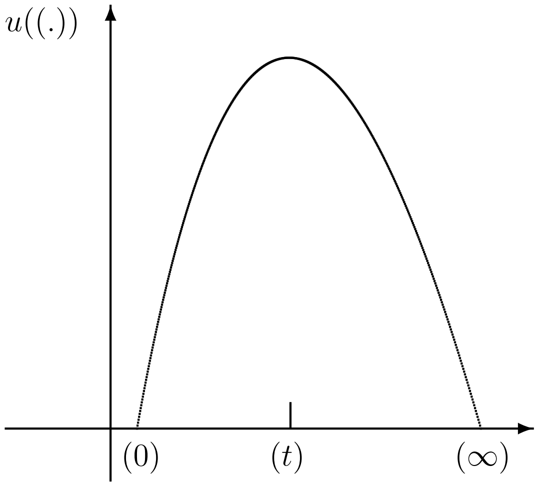
As in the previous section, we instrument our analysis using a function μ, depending on the primitives of the model. It defines a domination threshold between the strategy of facing difficulties since the beginning and the strategy of postponing them for one period, that is, between (0) and (1). For values of $q^{(0)}$ higher or equal than this threshold, (0) is preferred to (1) and for values of $q^{(0)}$ smaller than it, (1) is preferred to (0). This threshold and the stationary probabilities, determine optimal strategies. Let us first focus on the case in which assumption 2 does not play a role, that is, when DM's optimal strategy, in fact, belongs to the class of strategies already prescribed this assumption. We comment on the remaining cases afterwards. Results are as follows:

Theorem 2. *The DM's optimal strategy is to face difficulties since the beginning whenever she always enjoys the full capacity state with high enough probability (that is, whenever $\mu \leq q^d < q^e, q^{(0)}$), to always avoid them whenever she always experiences the full capacity state with low enough probability (that is, whenever $q^{(0)}, q^d < q^e \leq \mu$) and to face them from an intermediate point in time whenever she gets motivated through outcome achievements associated to easy tasks (that is, whenever $q^{(0)} < \mu \leq q^d < q^e$).*

In contrast with previous results, jumping into difficult tasks at some point in time can now be optimal. We interpret this strategy as one in which the DM prefers to first deal with easy tasks, because performing properly motivates her to deal with difficult but more rewarding tasks.

In the following figure we depict the ranking of utilities in this case. The DM exhibits single-peaked preferences on the optimal time to face difficulties, with the peak corresponding to an intermediate strategy:

Figure 3. The optimal strategy is (t)



The question of when to do the hard stuff arised in Quora, an internet knowledge market, in which people discuss about a specific given topic. The topic was: Is it better to do easy tasks first and then move on to harder ones, or vice versa?[15] One of the answers, that accurately illustrates our statement, was:

Important is to evaluate, which are the harder tasks and which the easy tasks. Out of this it becomes clear, how long it will take to do them. (...) The rest has more psychological character and is strongly depending on the personality. I personally like to mix it. This gives the success feeling, if you do the easy tasks and motivates, to continue with the harder tasks, to make the overall project the success.

If individuals indeed behave this way, there will be chances of improving achievements by dealing with motivation. Also, a model of human capital accumulation in which individuals build their skills by developing easy tasks up to the point that it is optimal for them to face difficulties, might offer the same type of results. However we truly think that the human capital accumulation story is essentially different from the motivational story. This difference relies on the following reasoning: while individuals build their human capital in the actual process of developing a task, motivation results when outcomes are achieved. We think that this is a crucial distinction, that might give different conclusions when deciding, for instance, the path of task difficulties with which individuals should be confronted. There might be a trade-off if easy task promote lower learning and high motivation due to more frequent good outcomes, and the contrary happens with difficult tasks.

We now briefly comment on some cases in which assumption 2 plays a role. That is, cases in which the DM has to choose the optimal strategy among the class of strategies prescribed by assumption 2, regardless of whether other path of action would have delivered higher utility. Under $q^d < q^e < \mu \leq q^{(0)}$ the DM would

[15]See http://www.quora.com/Is-it-better-to-do-easy-tasks-first-and-then-move-on-to-harder-ones-or-vice-versa.

have preferred to switch to easy tasks after have been dealing with difficulties for a while. Within the class of strategies she can choose among due to assumption 2, the DM exhibits single deep preferences on the optimal time to face difficulties. The deep corresponds to an intermediate strategy and the peaks correspond to the extreme strategies or either never dealing with difficulties or facing them since the beginning. The same happens under $q^d < \mu \leq q^e, q^{(0)}$. Among the available strategies prescribed by assumption 2, the DM ends up dealing with difficulties since the beginning. Finally under $q^{(0)}, q^d \leq \mu < q^e$ the DM would have also preferred to switch to easy tasks after have been dealing with difficulties for a while. As a result of assumption 2, the best thing the DM can do is to perform an intermediate strategy.[16]

The following result deals with the properties of the utility under the three optimal strategies. It also describes the returns of a boost in the DM's initial disposition, that is, $q^{(0)}$. We assume that a marginal increase in the initial disposition, does not affect the originally optimal strategy. Specifically, for the case in which an intermediate strategy is optimal we consider that marginal increase in the initial disposition, does not affect the particular point in time to face difficulties. Formally, when (t) is optimal then $t|_{q^{(0)}} = t|_{q^{(0)}+\epsilon}$ holds. Before stating the results let us denote by $mr((.))$, the marginal return of a increase in the DM's initial disposition. Proposition 2 is as follows:

Proposition 2. *The long-run expected utility is monotonically increasing and linear in $q^{(0)}$ under any optimal strategy. Moreover, $mr((0)) > mr((t-1)) > mr((t)) > mr((\infty))$.*

These results, as the ones in Proposition 1, capture the idea that individuals with better abilities perform better and achieve higher utility. It is also the case that advantaged individuals, those with high $q^{(0)}$, benefit more from a marginal increase in their abilities. As the table below illustrates, as $q^{(0)}$ increases within a row, everything else equal, that is, as the DM is of higher initial dispositions, she moves from finding (∞) or (t) optimal to (possibly) finding (0) optimal.

In the previous section we illustrated how the utility of high disposition individuals that always confront difficulties was higher than the utility of low disposition individuals that always avoid difficulties. We also carry such an analysis in this framework. We list in the table below the three combinations in which μ, and $q^e > q^d$ and $q^{(0)}$ relates to each other and the optimal strategies in every situation. Within each combination we consider that μ, q^e and q^d remain unaltered. However, they might be different across combinations. For the ease of exposition we only consider strict inequalities here:[17]

Table 2. Optimal strategies

[16] See the proof of Theorem 2 for a complete analysis.

[17] For a complete analysis, see the proof of Theorem 2

C.1	$q^{(0)} < q^d < q^e < \mu$ (∞)	$q^d < q^{(0)} < q^e < \mu$ (∞)	$q^d < q^e < q^{(0)} < \mu$ (∞)	$q^d < q^e < \mu < q^{(0)}$ (0) or (∞)
C.2	$q^{(0)} < \mu < q^d < q^e$ (t)	$\mu < q^{(0)} < q^d < q^e$ (0)	$\mu < q^d < q^{(0)} < q^e$ (0)	$\mu < q^d < q^e < q^{(0)}$ (0)
C.3	$q^{(0)} < q^d < \mu < q^e$ (t)	$q^d < q^{(0)} < \mu < q^e$ (t)	$q^d < \mu < q^{(0)} < q^e$ (0)	$q^d < \mu < q^e < q^{(0)}$ (0)

Let us focus on optimal strategies in the row corresponding to C.2, that is, either (t) or (0). In this case assumption 2 does not play a role. This allows us to make a neat comparison of the utility gains under the optimal strategies. The result is as follows:

Lemma 1. *Consider a DM characterized by $q^{(0)}$. The optimal strategy of facing difficulties since the beginning (that is, whenever $\mu < q^{(0)}, q^d < q^e$) yields higher utility than the optimal strategy of facing them from an intermediate point in time (that is, whenever $q^{(0)} < \mu < q^d < q^e$).*

Also, in order to make strategy (t) in C.2 and strategy (∞) in C.1 comparable, we consider, for the latter, the specific situation in which $q^{(0)} < q^d < q^e < \mu$. Notice that this is the only situation in C.1 in which $q^{(0)} < q^d$. Let $q^{(0)}$ be the same in both scenarios and focus on the case in which the only difference between C.1 and C.2 is that we increase μ, from C.2 to C.1, by decreasing K.[18] Since q^e and q^d do not depend on K, they remain unaltered. The result is as follows

Lemma 2. *Consider a DM characterized by $q^{(0)}$. The optimal strategy of facing difficulties from an intermediate point in time (that is, whenever $q^{(0)} < \mu < q^d < q^e$) yields higher utility than the optimal strategy of always avoiding difficulties (that is, whenever $q^{(0)} < q^d < q^e < \mu$).*

With these two lemmas we conclude that optimal strategies involving the choice of difficulties at some point in time, yield higher utility than optimal strategies in which the DM always avoids difficulties.

4 Conclusions

Non-cognitive abilities have an impact in determining performance in dimensions of huge economic relevance, as labor market entry/ search decisions or educational attainments. We link, in a dynamic setting, non-cognitive abilities to the decision of when to deal with difficult but valuable tasks. We show how low disposition individuals always avoid difficulties and forego better economic opportunities while high disposition individuals are willing to deal with difficulties. The behavior of individuals that always avoid dealing with onerous tasks resembles procrastination results,

[18] See the proof of Theorem 2, step 1.

without relying on the hyperbolic discounting assumption.[19] Also, individuals that get motivated by outcome achievements find optimal to jump into difficult tasks at some point in time.

5 Appendix. Proofs

Before proceeding we set some useful definitions. Let us denote by $u^{1,(0)}$ and $u^{2,(0)}$, the DM's long-run expected utility when she only experiences the deteriorated capacity state s_1, that is when $q = 0$, and when she only experiences the full capacity state s_2, that is when $q = 1$, respectively, under strategy (0). Similarly, let us denote by $u^{1,(1)}$ and $u^{2,(1)}$, the DM's long-run expected utility when she only experiences the deteriorated capacity state s_1, that is when $q = 0$, and when she only experiences the full capacity state s_2, that is when $q = 1$, respectively, under strategy (1).

Let us define functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ as $f(\lambda) = \lambda u^{2,(1)} + (1-\lambda)u^{1,(1)}$ and $g(\lambda) = \lambda u^{2,(0)} + (1-\lambda)u^{1,(0)}$, respectively. For $\lambda \in [0, 1]$, these functions are the convex combination of the DM's long-run expected utilities, when she experiences s_2 with probability $q = 1$ and $q = 0$, out of strategies (1) and (0), respectively. Furthermore, we say that a strategy (t) dominates strategy $(t + 1)$ whenever the long-run expected utility of (t) is higher than the one of $(t + 1)$. Let $(t) > (t + 1)$ denote this domination relationship. Recall that (t) denotes any strategy such that $0 < t < \infty$. We also say that strategy (0) dominates strategy (1) whenever the long-run expected utility of (0) is higher than the one of (1). Let $(0) > (1)$ denote this domination relationship.

Proof of Theorem 1. The proof is composed by two steps. In Step 1 we derive the threshold λ such $f(\lambda)$ and $g(\lambda)$ equalize. For such a λ, (1) and (0) yield the same long-run expected utility. For values higher or equal than λ then $(0) > (1)$. For values lower than λ then $(1) > (0)$. In step 2 we argue how this information is enough to set the optimal strategy, depending on the values of λ and q.

Step 1. If the DM experiences s_2 with probability $q = 1$, the long-run expected utility of strategy (0) is $u^{2,(0)} = Kp_{22}+\delta u^{2,(0)}$. If she experiences s_1 with probability $1 - q = 1$, the long-run expected utility of strategy (0) is $u^{1,(0)} = Kp_{12} + \delta u^{1,(0)}$. Similarly, when she experiences s_2 with probability $q = 1$, the long-run expected utility of strategy (1) is $u^{2,(1)} = p_{21} + \delta u^{2,(0)}$ whereas when she experiences s_1 with probability $1 - q = 1$, then $u^{1,(1)} = p_{11} + \delta u^{1,(0)}$. From previous definitions, $f(\lambda) = \lambda(u^{2,(1)} - u^{1,(1)}) + u^{1,(1)}$ and $g(\lambda) = \lambda(u^{2,(0)} - u^{1,(0)}) + u^{1,(0)}$. Solving $f(\lambda) = g(\lambda)$ for λ we get $\lambda((u^{2,(1)} - u^{1,(1)}) - (u^{2,(0)} - u^{1,(0)})) = u^{1,(0)} - u^{1,(1)}$. Notice that $u^{2,(0)} - u^{1,(0)} = K(p_{22} - p_{12})(1-\delta)^{-1}$ and $u^{1,(0)} = Kp_{12}(1-\delta)^{-1}$. Thus, $\lambda((p_{21} - p_{11}) - K(p_{22} - p_{12})) = Kp_{12} - p_{11}$ or $\lambda = (Kp_{12} - p_{11})((p_{21} - p_{11}) - K(p_{22} - p_{12}))^{-1}$. Assumption 1 implies that $p_{22} - p_{12} > p_{21} - p_{11}$, thus the denominator of the previous

[19] See (Rubinstein, 2003) for a discussion on this assumption.

is different from zero, and λ exists. Also the denominator is negative and for values lower (respectively higher) than λ then $(1) > (0)$ (respectively $(0) > (1)$). For values equal to λ we assume that $(0) > (1)$ as well. Assumption 1 also implies that $p_{11}/p_{12} > p_{21}/p_{22}$.[20] Thus, $\partial\lambda/\partial K = \dfrac{p_{21}p_{12} - p_{22}p_{11}}{((p_{21} - p_{11}) - K(p_{22} - p_{12}))^2} < 0$. Notice than when $K = p_{21}/p_{22}$ then $\lambda = 1$ and when $K = p_{11}/p_{12}$ then $\lambda = 0$. It then follows that when $K < p_{21}/p_{22}$ then $\lambda > 1$, and no matter q, $(1) > (0)$. On the contrary, when $K > p_{11}/p_{12}$ then $\lambda < 0$, and no matter q, $(0) > (1)$. We focus on the interesting case such that $p_{21}/p_{22} < K < p_{11}/p_{12}$ and $\lambda \in (0, 1)$. We thus conclude that when $q \geq \lambda$ then $(0) > (1)$ and when $q < \lambda$ then $(1) > (0)$.

Step 2. We set here the optimal strategies. Two cases arise depending on the relation between q and λ:

C.1. Suppose that $q \geq \lambda$. By step 1, $(0) > (1)$. Let us compare any pair of intermediate strategies (t) and $(t+1)$. We have that $u((t)) = \sum_{i=0}^{t-1} \delta^i(qp_{21} + (1 - q)p_{11}) + K\sum_{i=t}^{\infty} \delta^i(qp_{22} + (1 - q)p_{12})$ and $u((t + 1)) = \sum_{i=0}^{t-1} \delta^i(qp_{21} + (1 - q)p_{11}) + \delta^t(qp_{21} + (1 - q)p_{11}) + K\sum_{i=t+1}^{\infty} \delta^i(qp_{22} + (1 - q)p_{12})$. Notice that up to the point in time $t - 1$, (t) and $(t + 1)$ yield the same utility. Notice also that from time t on, the comparison is between (0) and (1), evaluated from the point of view of time t. Since it is always the case that $q \geq \lambda$, it follows that $(0) > (1)$, from the point of view of time t. That is so because we can consider the process as starting at time t and thus, apply step 1. As a consequence, for any pair of intermediate strategies, (t) and $(t+1)$, it follows that $(t) > (t+1)$. It is useful to observe that $\lim_{i\longrightarrow\infty} u((t+i)) = u((\infty))$. We then conclude that $(0) > (1) > ... > (t) > (t + 1) > ... > (\infty)$ holds. In this case (0) is optimal.

C.2. Suppose that $q < \lambda$. To conclude that $(0) < (1) < ... < (t) < (t+1) < ... < (\infty)$ we use a similar reasoning as above and thus omit it here. In this case (∞) is optimal.

■

Proof of the Remark. See the proof of Theorem 1 for the expression of λ and its relation with K. We analyze here how λ varies with the probabilities of success. Let $x \equiv (p_{21} - p_{11} - K(p_{22} - p_{12}))^2$ be the denominator in the following derivatives. We have that: $\partial\lambda/\partial p_{11} = (Kp_{22} - p_{21})x^{-1}$, $\partial\lambda/\partial p_{12} = K(p_{21} - Kp_{22})x^{-1}$, $\partial\lambda/\partial p_{21} = (p_{11} - Kp_{12})x^{-1}$ and $\partial\lambda/\partial p_{22} = K(Kp_{12} - p_{11})x^{-1}$. Since $K \in (p_{21}/p_{22}, p_{11}/p_{12})$ then $\partial\lambda/\partial p_{i1} > 0$ and $\partial\lambda/\partial p_{i2} < 0$ with $i = 1, 2$.

■

Proof of Proposition 1. Recall that $u((0)) = K\sum_{i=0}^{\infty} \delta^i(qp_{22}+(1-q)p_{12}) = (K(qp_{22}+(1 - q)p_{12})(1 - \delta)^{-1}$ and $u((\infty)) = \sum_{i=0}^{\infty} \delta^i(qp_{21} + (1 - q)p_{11}) = (qp_{21} + (1 - q)p_{11})(1 - \delta)^{-1}$. Since $\partial u((0))/\partial q = K(p_{22} - p_{12})(1 - \delta)^{-1} > 0$ and $\partial u((\infty))/\partial q = (p_{21} - p_{11})(1 - \delta)^{-1} > 0$, utilities are increasing and linear in q. Finally, assumption

[20] By assumption 1, $p_{11} - p_{12} > p_{21} - p_{22}$. It implies that $(p_{11} - p_{12})(p_{12})^{-1} > (p_{21} - p_{22})(p_{22})^{-1}$ or equivalently $p_{11}/p_{12} > p_{21}/p_{22}$.

1 implies that $p_{22} - p_{12} > p_{21} - p_{11}$, then the marginal return of an increase in q is the highest under (0). ■

Before the proof of Theorem 2 let us set two useful claims:

Claim 1. *Consider that the DM repeatedly faces easy tasks. Then, at every time t, $q^{(0)} > q^{(t)} > q^{(t+1)} > q^e$ whenever $q^{(0)} > q^e$ and $q^{(0)} < q^{(t)} < q^{(t+1)} < q^e$ whenever $q^{(0)} < q^e$.*

Proof of Claim 1. The proof is by induction. Let us focus on the case in which $q^{(0)} > q^e$. We first prove that for $t = 1$, $q^{(0)} > q^{(1)} > q^e$ holds. We set the induction part afterwards.

Step 1. $q^{(0)} > q^{(1)} > q^e$. In showing that $q^{(0)} > q^{(1)}$, we compare the initial probability of experiencing s_2, with its first perturbation, after having decided to face an easy task. Consider expression (1) in the main body:

$$[q^{(0)} \quad 1-q^{(0)}] \begin{bmatrix} p_{21} & 1-p_{21} \\ p_{11} & 1-p_{11} \end{bmatrix} = [q^{(1)} \quad 1-q^{(1)}].$$

We have that $q^{(1)} = q^{(0)}p_{21} + (1-q^{(0)})p_{11}$. Recall that $q^{(0)} > q^e$ and $q^e = p_{11}(1-p_{21}+p_{11})^{-1}$. Thus, $q^{(0)} > p_{11}(1-p_{21}+p_{11})^{-1}$ or equivalently $q^{(0)}(1-p_{21}+p_{11}) > p_{11}$. We rewrite this expression as $q^{(0)} > q^{(0)}p_{21}+(1-q^{(0)})p_{11}$. The RHS of this expression is exactly $q^{(1)}$. In showing that $q^{(1)} > q^e$ we proceed by contradiction. Suppose that $q^{(1)} < q^e$ holds, that is, $q^{(0)}p_{21} + (1-q^{(0)})p_{11} < q^e$. This is equivalent to $q^{(0)}p_{21}+(1-q^{(0)})p_{11} < p_{11}(1-p_{21}+p_{11})^{-1}$ or $(p_{11}+q^{(0)}(p_{21}-p_{11}))(1-p_{21}+p_{11}) < p_{11}$. Rearranging terms it becomes $p_{11}-p_{11}(p_{21}-p_{11})+q^{(0)}(p_{21}-p_{11})(1-p_{21}+p_{11}) < p_{11}$. This is equivalent to $q^{(0)} < p_{11}(1-p_{21}+p_{11})^{-1} = q^e$, contradicting our initial assumption. Thus, $q^{(0)} > q^{(1)} > q^e$ holds.

Step 2. If for an arbitrary t, $q^{(t)} > q^{(t+1)} > q^e$ holds, for $q^{(t+1)}$ we have:

$$\left[q^{(t+1)} \quad 1-q^{(t+1)}\right] \begin{bmatrix} p_{21} & 1-p_{21} \\ p_{11} & 1-p_{11} \end{bmatrix} = [q^{(t+2)} \quad 1-q^{(t+2)}].$$

In concluding that $q^{(t+1)} > q^{(t+2)} > q^e$ we use exactly the same reasoning than in the previous step. We conclude that $q^{(0)} > q^{(t)} > q^{(t+1)} > ... > q^e$ holds. The case in which $q^{(0)} < q^e$ relies on the same argument. The same analysis goes through for describing the relation between $q^{(t)}$ and q^d. We thus omit the proofs here. ■

Claim 2. $q^{(i)} = q^{(0)}(T^k)^i + p_{1k}\sum_{j=0}^{i-1}(T^k)^j$ *and* $q^{(t+i)} = q^{(t)}(T^k)^i + p_{1k}\sum_{j=0}^{i-1}(T^k)^j$ *with $k = e, d$.*

Proof of Claim 2. Recall that $T^d = p_{22} - p_{12}$ and $T^e = p_{21} - p_{11}$. By expression (1) in the main body, $q^1 = q^{(0)}T^k + p_{1k}$. Also $q^2 = q^{(1)}T^k + p_{1k} = (q^{(0)}T^k + p_{1k})T^k + p_{1k} = q^{(0)}(T^k)^2 + p_{1k}T^k + p_{1k}$. In general $q^{(i)} = q^{(0)}(T^k)^i + p_{1k}(T^k)^{i-1} + ... + p_{1k}T^k + p_{1k}$ or $q^{(i)} = q^{(0)}(T^k)^i + p_{11}\sum_{j=0}^{t-1}(T^k)^j$. To conclude that $q^{(t+i)} = q^{(t)}(T^k)^i + p_{1k}\sum_{j=0}^{i-1}(T^k)^j$ we follow a similar reasoning. We thus omit it here. ■

Proof of Theorem 2. We follow the same steps than in the proof of Theorem 1.

Step 1. Consider expression (1) in the main body. When the DM experiences s_2 with probability $q^{(0)} = 1$, the long-run expected utility out of strategy (0) is $u^{2,(0)} = Kp_{22} + \delta(p_{22}u^{2,(0)} + (1-p_{22})u^{1,(0)})$. When she experiences s_1 with probability $1-q^{(0)} = 1$, the long-run expected utility of strategy (0) is $u^{1,(0)} = Kp_{12} + \delta(p_{12}u^{2,(0)} + (1-p_{12})u^{1,(0)})$. Solving for $u^{1,(0)}$ and $u^{2,(0)}$ we get $u^{2,(0)} = \dfrac{K(p_{22}-\delta T^d)}{(1-\delta)(1-\delta T^d)}$ and that $u^{1,(0)} = \dfrac{Kp_{12}}{(1-\delta)(1-\delta T^d)}$. Thus, $u^{2,(0)} - u^{1,(0)} = \dfrac{KT^d}{1-\delta T^d}$. When she experiences s_2 with probability $q^{(0)} = 1$, the long-run expected utility out of strategy (1) is $u^{2,(1)} = p_{21} + \delta(p_{21}u^{2,(0)} + (1-p_{21})u^{1,(0)})$. Similarly, when she experiences s_1, with probability $1-q^{(0)} = 1$, the long-run expected utility out of strategy (1) can is $u^{1,(1)} = p_{11} + \delta(p_{11}u^{2,(0)} + (1-p_{11})u^{1,(0)})$. Now, $f(\mu) = \mu(u^{2,(1)} - u^{1,(1)}) + u^{1,(1)}$ and $g(\mu) = \mu(u^{2,(0)} - u^{1,(0)}) + u^{1,(0)}$ Solving for μ such that $f(\mu) = g(\mu)$ we get $\mu = \dfrac{u^{1,(0)} - u^{1,(1)}}{(u^{2,(1)} - u^{1,(1)}) - (u^{2,(0)} - u^{1,(0)})}$. With respect to the numerator, $u^{1,(0)} - u^{1,(1)} = (1-\delta)u^{1,(0)} - p_{11} - \delta p_{11}(u^{2,(0)} - u^{1,(0)})$ or equivalently $u^{1,(0)} - u^{1,(1)} = \dfrac{(1-\delta)Kp_{12}}{(1-\delta)(1-\delta T^d)} - p_{11} - \dfrac{\delta p_{11}KT^d}{1-\delta T^d} = \dfrac{K(p_{12}-\delta p_{11}T^d) - (p_{11}-\delta p_{11}T^d)}{1-\delta T^d}$. Regarding the denominator, $u^{2,(1)} - u^{1,(1)} = T^e + \delta T^e(u^{2,(0)} - u^{1,(0)})$ and $u^{2,(1)} - u^{1,(1)} - (u^{2,(0)} - u^{1,(0)}) = T^e - (1-\delta T^e)(u^{2,(0)} - u^{1,(0)})$. This is equivalent to $\dfrac{T^e(1-\delta T^d) - KT^d(1-\delta T^e)}{(1-\delta T^d)}$. Thus, $\mu = \dfrac{K(p_{12}-\delta p_{11}T^d) - (p_{11}-\delta p_{11}T^d)}{T^e(1-\delta T^d) - KT^d(1-\delta T^e)}$.[21] Since by assumption 1, $T^d > T^e$ the denominator is different from zero, hence μ is a real number. Since by assumption 1, the denominator is negative, for values lower than μ then (1) > (0) and for values higher than it, (0) > (1). For values equal to μ we assume that (0) > (1) as well. Also by assumption 1, $\partial\mu/\partial K = \dfrac{(1-\delta T^d)(p_{21}p_{12} - p_{11}p_{22})}{(T^e(1-\delta T^d) - KT^d(1-\delta T^e))^2} < 0$. Furthermore, when $K = \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$ then $\mu = 0$ and when $K = \dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}$ then $\mu = 1$. Thus, it has to be that $\dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d} < \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$.[22] It also has to be that when $K > \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-\delta p_{11}T^d}$ then $\mu < 0$. In this case no matter $q^{(0)}$, (0) > (1). In contrast, when $K < \dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}$, then $\mu > 1$ and no matter $q^{(0)}$, (1) > (0). The interesting case is such that $K \in \left(\dfrac{p_{21}-\delta p_{21}T^d}{p_{22}-\delta p_{21}T^d}, \dfrac{p_{11}-\delta p_{11}T^d}{p_{12}-p_{11}\delta T^d}\right)$ and $\mu \in (0,1)$. We conclude that when $q^{(0)} \geq \mu$

[21]When the DM does not care about the future, that is, when $\delta = 0$, $\lambda = \mu$. That q does not vary over time is conceptually equivalent to think about a DM making one period decisions without consequences on her subsequent states. Additionally, $\partial\mu/\partial\delta = K(K-1)(p_{11}p_{22} - p_{21}p_{22}) > 0$, meaning that the more the DM cares about the future the more she postpones difficult tasks, where good outcomes are less frequent.

[22]Notice that since we analyze the case in which K is equal to either part of the inequality, they have to be positive. In fact the LHS is always positive and higher than 1. However the RHS might be negative due to the denominator, in this case it is immaterial as an upper bound.

then $(0) > (1)$ and when $q^{(0)} < \mu$ then $(1) > (0)$.

Step 2. We set here the optimal strategies. Consider first, that $\mu \leq q^{(0)}$. By step 1, $(0) > (1)$ from the point of view of $q^{(0)}$. Three cases arise:

C.1. Suppose that $\mu \leq q^d < q^e, q^{(0)}$. Let us compare any pair of intermediate strategies, (t) and $(t+1)$. We thus evaluate $u((t)) = \sum_{i=0}^{t-1}(q^{(i)}p_{21} + (1-q^{(i)})p_{11}) + K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$ versus $u((t+1)) = \sum_{i=0}^{t-1}(q^{(i)}p_{21} + (1-q^{(i)})p_{11}) + \delta^t(q^{(t)}p_{21} + (1-q^{(t)})p_{11}) + K\sum_{i=t+1}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. Notice that up to time $t-1$, both expressions yield the same utility. From time t on, the comparison is between (0) and (1), from the point of view of $q^{(t)}$. Notice that for any strategy (t), $q^{(t)}$ results from have been dealing with easy tasks up to time $t-1$. Thus, by Claim 1, $q^{(t)} > \mu$. This implies that, from the point of view of $q^{(t)}$, $(0) > (1)$. That is so because we can consider the process as starting at time t, and thus apply step 1. As a consequence, for any pair of intermediate strategies (t) and $(t+1)$, it follows that $(t) > (t+1)$. Recall that $\lim_{i\longrightarrow\infty} u((t+i)) = u((\infty))$.[23] We thus establish that $(0) > (1)... > (t) > (t+1) > ... > (\infty)$. Then (0) is optimal.

Assumption 2 does not play any role in C.1, that is, the DM's optimal strategy is within the class of strategies that it prescribes. However, it does in C.2 and C.3 below. In both cases the DM would have found optimal to start with difficult tasks and to switch to easy ones at some point in time. We look for the optimal strategies within the ones prescribed by assumption 2.

C.2. Suppose that $q^d < \mu \leq q^e, q^{(0)}$. Let us compare (t) and $(t+1)$, as above. We then evaluate $u((t))$ and $u((t+1))$ as defined in $C.1$. The relevant comparison is between (0) and (1) from the point of view of $q^{(t)}$. Notice that $q^{(t)}$ results from dealing with easy tasks up to time $t-1$. Thus, by Claim 1, $q^{(t)} \geq \mu$. Then, by step 1, from the point of view of $q^{(t)}$, $(0) > (1)$ and, as a consequence, $(t) > (t+1)$. We thus establish that $(0) > (1) > ... > (t) > (t+1) > ... > (\infty)$, being (0) optimal.

C.3. Suppose that $q^d < q^e < \mu \leq q^{(0)}$. Let us compare (t) and $(t+1)$ as above. We then evaluate $u((t))$ and $u((t+1))$ as defined in C.1. The relevant comparison is between (0) and (1) from the point of view of $q^{(t)}$. Suppose that strategy (t) is such that $q^{(t)} > \mu$. Again, from the point of view of $q^{(t)}$, $(0) > (1)$. As a consequence, $(t) > (t+1)$. Suppose that strategy (t^*-1) is such that $q^{(t^*-1)} = \mu$.[24] Notice that $t < t^* - 1$, by Claim 1 and, at least, $t^* - 1 = t + 1$. Thus, from the point of view of $q^{(t^*-1)}$, $(0) > (1)$. As a consequence, $(t^*-1) > (t^*)$. Suppose that strategy (t) is such that $q^{(t)} < \mu$ for the first time. By Claim 1, this point in time has to be exactly t^*. Thus, from the point of view of $q^{(t^*)}$, $(1) > (0)$. As a consequence $(t^*+1) > (t^*)$. Suppose that strategy (t) is any is such that $q^{(t)} < \mu$. Notice that $t^* < t$, by Claim 1 and, at least, $t = t^* + 1$. Thus, from the point of view of $q^{(t)}$, $(1) > (0)$. As a consequence, $(t) < (t+1)$, in particular, $(t^*+1) < (t^*+2)$. In general we have that $(0) > (1) > ... > (t^*-1) > (t^*) < (t^*+1) < (t^*+2) < ... < (\infty)$. An intermediate

[23] This observation applies to the remaining cases. We omit it in what follows.

[24] The same analysis follows if we consider that $q^{(t^*-1)} < \mu$.

strategy (t) is then the least preferred and the optimal is either (0) or (∞).

Consider now that $q^{(0)} < \mu$. By step 1, (1) > (0) from the point of view of $q^{(0)}$. Three cases arise. As we use parallel arguments than above, we go briefly over them. We compare $u((t))$ and $u((t+1))$ in every case. The relevant comparison will be between (0) and (1), from the point of view of $q^{(t)}$:

C.1. Suppose that $q^{(0)}, q^d < q^e \leq \mu$. Notice that for any strategy (t), $q^{(t)}$ results from have been dealing with easy tasks up to time $t-1$. Thus, by Claim 1, $q^{(t)} < \mu$. Thus by step 1, (1) > (0) from the point of view of $q^{(t)}$. As a consequence, for any pair of strategies, $(t+1) > (t)$. Thus, $(0) < (1) < ... < (t) < (t+1) < ... < (\infty)$ and the optimal strategy is (∞).

C.2. Suppose that $q^{(0)} < \mu \leq q^d < q^e$. Suppose that strategy (t) is such that $q^{(t)} < \mu$. Thus, from the point of view of $q^{(t)}$, (1) > (0). As a consequence, $(t+1) > (t)$. Suppose that strategy (t^*) is such that $q^{(t^*)} = \mu$. Notice that $t < t^*$, by Claim 1 and, at least, $t^* = t+1$. Thus, from the point of view of $q^{(t^*)}$, (0) > (1). As a consequence, $(t^*) > (t^*+1)$. Suppose that strategy (t) is such that $q^{(t)} > \mu$ for the first time. By Claim 1, this point in time has to be exactly t^*+1. Thus, from the point of view of $q^{(t^*+1)}$, (0) > (1). As a consequence $(t^*+1) > (t^*+2)$. Suppose that strategy (t) is any other such that $q^{(t)} > \mu$. Notice that $t^*+1 < t$, by Claim 1 and, at least, $t = t^*+2$. Thus, from the point of view of $q^{(t)}$, (0) > (1). As a consequence $(t) > (t+1)$. We thus have that, $(0) < (1) < ... < (t^*-1) < (t^*) > (t^*+1) > (t^*+2) > ... > (\infty)$. Then, (t) is optimal.

Assumption 2 does not play a role in $C.1$ and $C.2$. However, it does in the last case. In it, the DM would have preferred to switch from difficult to easy tasks at some point. We look for the optimal strategies within the ones prescribed by assumption 2.

C.3. Suppose that $q^{(0)}, q^d \leq \mu < q^e$. Suppose that strategy (t) is such that $q^{(t)} < \mu$. Thus, from the point of view of $q^{(t)}$, (1) > (0). As a consequence, $(t+1) > (t)$. Suppose that strategy (t^*) is such that $q^{(t^*)} = \mu$. Notice that $t < t^*$, by Claim 1 and, at least, $t^* = t+1$. Thus, from the point of view of $q^{(t^*)}$, (0) > (1). As a consequence, $(t^*) > (t^*+1)$. Suppose that strategy (t) is such that $q^{(t)} > \mu$ for the first time. This point in time is exactly t^*+1. Thus, from the point of view of $q^{(t^*+1)}$, (0) > (1). As a consequence $(t^*+1) > (t^*+2)$. Suppose that strategy (t) is any strategy such that $q^{(t)} > \mu$. Notice that $t^*+1 < t$, by Claim 1 and, at least, $t = t^*+2$. Thus, from the point of view of $q^{(t)}$, (0) > (1). As a consequence, $(t) > (t+1)$. Summing up we have that $(0) < (1) < ... < (t^*-1) < (t^*) > (t^*+1) > (t^*+2) > ... > (\infty)$. Then, (t) is optimal.

■

Proof of Proposition 2. We have three cases depending on the optimal strategy:

C.1.(0) is optimal. We have that $u((0)) = K\sum_{i=0}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$.

By Claim 2, $q^{(i)} = q^{(0)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j$. Plugging $q^{(i)}$ in the previous expression we have that $u((0)) = K\sum_{i=0}^{\infty}\delta^i(q^{(i)}T^d + p_{12})$ or $KT^d q^{(0)}\sum_{i=0}^{\infty}(\delta T^d)^i + p_{12}T^d K\sum_{i=0}^{\infty}\delta^i\sum_{j=0}^{i-1}(T^d)^j + Kp_{12}\sum_{i=0}^{\infty}\delta^i$. Then $\partial u((0))/\partial q^{(0)} = KT^d(1-\delta T^d)^{-1} > 0$.

C.2. (∞) is optimal. We have that $u((\infty)) = \sum_{i=0}^{\infty}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11})$. We follow exactly the same reasoning than in C.1, and thus omit it here. In this case $\partial u((\infty))/\partial q^{(0)} = T^e(1-\delta T^e)^{-1} > 0$.

In both cases utility is increasing and linear in $q^{(0)}$. Since $T^d > T^e$, the marginal return of an increase in $q^{(0)}$ is higher under (0) than under (∞).

C.3. (t) is optimal. We have that $u((t)) = \sum_{i=0}^{t-1}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11}) + K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. Let us focus first on the first part of the expression, that is, $\sum_{i=0}^{t-1}\delta^i(q^{(i)}p_{21} + (1-q^{(i)})p_{11})$. By Claim 2, $q^{(i)} = q^{(0)}(T^e)^i + p_{11}\sum_{j=0}^{i-1}(T^e)^j$. Thus, $\sum_{i=0}^{t-1}\delta^i(q^{(i)}T^e + p_{11}) = \sum_{i=0}^{t-1}\delta^i((q^{(0)}(T^e)^i + p_{11}\sum_{j=0}^{i-1}(T^e)^j)T^e + p_{11})$. This expression is equivalent to $q^{(0)}T^e\sum_{i=0}^{t-1}\delta^i(T^e)^i + T^e\sum_{i=0}^{t-1}\delta^i p_{11}\sum_{j=0}^{i-1}(T^e)^j + \sum_{i=0}^{t-1}\delta^i p_{11}$. Its derivative with respect to $q^{(0)}$ is $T^e\sum_{i=0}^{t-1}(\delta T^e)^i > 0$. Consider now the second part, that is, $K\sum_{i=t}^{\infty}\delta^i(q^{(i)}p_{22} + (1-q^{(i)})p_{12})$. By Claim 2, $q^{(t+i)} = q^{(t)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j$. Thus, we rewrite the previous expression as $K(\sum_{i=0}^{\infty}\delta^{t+i}((q^{(t)}(T^d)^i + p_{12}\sum_{j=0}^{i-1}(T^d)^j)T^d + p_{12}))$. This is equivalently rewritten as $K(q^{(t)}T^d\sum_{i=0}^{\infty}\delta^{t+i}(T^d)^i + T^d\sum_{i=0}^{\infty}\delta^{t+i}p_{12}\sum_{j=0}^{\infty}(T^d)^j + \sum_{i=0}^{\infty}\delta^{t+i}p_{12}))$. By Claim 2 we express the part depending on $q^{(t)}$ as $(q^{(0)}(T^d)^t + p_{12}\sum_{i=0}^{t-1}(T^d)^i)K\delta^t T^d(1-\delta T^d)^{-1}$. Taking derivatives w.r.t $q^{(0)}$ we get $K\delta^t(T^d)^{t+1}(1-\delta T^d)^{-1} > 0$. Summing up, we have that $\partial u((t))/\partial q^{(0)} = T^e\sum_{i=0}^{t-1}(\delta T^e)^i + K\delta^t((T^d)^{t+1})(1-\delta T^d)^{-1} > 0$.

We now compare the return of a marginal increase in $q^{(0)}$, in the aforementioned strategies. Notice that $u((0)) = K(\sum_{i=0}^{t-1}\delta^i(q^{(i)}T^d + p_{12}) + \sum_{i=t}^{\infty}\delta^i(q^{(i)}T^d + p_{12}))$ and $u((\infty)) = \sum_{i=0}^{t-1}\delta^i(q^{(i)}T^e + p_{11}) + \sum_{i=t}^{\infty}\delta^i(q^{(i)}T^e + p_{11})$. We use similar algebra as above to conclude that $\partial u((0))/\partial q^{(0)} = KT^d\sum_{i=0}^{t-1}(\delta T^d)^i + K\delta^t(T^d)^{t+1}(1-\delta T^d)^{-1}$ and $\partial u((\infty))/\partial q^{(0)} = T^e\sum_{i=0}^{t-1}(\delta T^e)^i + \delta^t(T^e)^{t+1}(1-\delta T^e)^{-1}$ Since $T^d > T^e$, we have that $\partial u((0))/\partial q^{(0)} > \partial u((t))/\partial q^{(0)}$ and $\partial u((t))/\partial q^{(0)} > \partial u((\infty))/\partial q^{(0)}$. We also compare the marginal return of any pair of intermediate strategies $(t-1)$ and (t). In this case $t-1 \geq 1$. We have that $\partial u((t-1))/\partial q^{(0)} = T^e\sum_{i=0}^{t-2}(\delta T^e)^i + K\delta^{t-1}(T^d)^t(1-\delta T^d)^{-1}$ and $\partial u((t))/\partial q^{(0)} = T^e\sum_{i=0}^{t-1}(\delta T^e)^i + K\delta^t(T^d)^{t+1}(1-\delta T^d)^{-1}$. The latter expression minus the former yields $\delta^{t-1}(K(T^d)^t - (T^e)^t) > 0$, since $T^d > T^e$. ■

Proof of Lemma 1. Consider C.2 in table 2 in the main body. Let us denote by $q^{(0)'}$ the initial probability in any of the cases in which (0) is optimal. Let us also denote by $q^{(0)}$ the initial probability in the case in which (t) is optimal. Notice that $q^{(0)'} > q^{(0)}$. Consider the utility of (t) when the DM is characterized by $q^{(0)'}$, that is, when (0) is optimal. Notice that the utility of (t) is higher when the DM is characterized by $q^{(0)'}$ than when she is characterized by $q^{(0)}$ and precisely (t) is optimal. To see this, notice that up to $t-1$ the DM faces easy tasks. Since

$q^{(0)'} > q^{(0)}$, by Claim 2, $q^{(i)'} > q^{(i)}$ at every $i \leq t-1$. Consider now points in time $i \geq t$. By Claim 1, $q^{(i)}$ approaches q^d from below without exceeding it. Also, $q^{(i)'}$ may approach q^d from below or above, without exceeding it.[25] When $q^{(i)'}$ approaches q^d from below, by Claim 2, $q^{(i)'} > q^{(i)}$. When $q^{(i)'}$ approaches q^d from above by Claim 1, $q^{(i)'} > q^d > q^{(i)}$. Since current expected utility at every time t, that is, $K(q^{(t)}p_{22} + (1 - q^{(t)}p_{12})$, is increasing $q^{(t)}$, it has to be that $u((t))$ is higher under $q^{(0)'}$ than under $q^{(0)}$. Also, under $q^{(0)'}$, $u((0)) > u((t))$ by optimality. We thus conclude that the optimal strategy (0) yields higher utility than the optimal strategy (t).

■

Proof of Lemma 2. Consider that the DM is characterized by $q^{(0)}$. By the proof of Theorem 2, under $q^{(0)} < q^d < q^e < \mu'$, (∞) is optimal whereas under $q^{(0)} < \mu < q^d < q^e$, (t) is optimal. Recall that μ is decreasing in K, thus $\mu < \mu'$ is associated to $K > K'$. By optimality of (t) we have that $\sum_{i=0}^{t-1} \delta^i (q^{(i)}T^e + p_{11}) + K \sum_{i=t}^{\infty} \delta^i (q^{(i)}T^d + p_{12}) > \sum_{i=0}^{t-1} \delta^i (q^{(i)}T^e + p_{11}) + \sum_{i=t}^{\infty} \delta^i (q^{(i)}T^e + p_{11})$. Since we consider the case in which $q^{(0)}$ as well as probabilities of success affecting q^d and q^e are the same, the RHS of this expression brings exactly the same utility that when $q^{(0)} < q^d < q^e < \mu'$, and hence (∞) is optimal. Thus, $u((t))$ under $q^{(0)} < \mu < q^d < q^e$ is higher than $u((\infty))$ under $q^{(0)} < q^d < q^e < \mu'$.

■

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[25] For Claim 1 to apply we consider, as in previous proofs, the process as starting at time $i = t$. Also, the behavior of $q^{(i)'}$ depends on whether in approaching q^e, q^d is exceeded or not.

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