PERSPECTIVAS

Revista de Análisis de Economía, Comercio y Negocios Internacionales

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Sumario

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Presentación

"PERSPECTIVAS: Revista de Análisis de Economía, Comercio y Negocios Internacionales", es un publicación semestral cuyo objetivo principal es la difusión de artículos académicos de alto rigor teórico y metodológico, abarcando temas sobre distintos campos de la teoría económica, el comercio y negocios; permitiéndose, también, la divulgación de artículos de discusión y aplicaciones que enriquezcan el pensamiento económico y/o contribuyan a la consolidación de la utilización de técnicas económicas en el entorno actual. Su misión es ser referencia para investigadores, estudiantes e interesados en cuanto a temas contemporáneos y discusiones actuales en la economía, así como crear un espacio para dar la bienvenida a autores de los sectores público y privado con el fin de vincular el estudio y la práctica de esta disciplina.

La cobertura temática de la revista es multidisciplinaria, en cuanto a los ejes fundamentales que se mencionan en el título, aunque principalmente se enfoca en las siguientes áreas:

- Microeconomía teórica y aplicada.
- Macroeconomía teórica y aplicada.
- Econometría.
- Teoría económica.
- Economía internacional.
- Matemática económica (Teoría de juegos, economía dinámica, optimización).
- Finanzas.
- Comercio internacional.
- Regulaciones internacionales.
- Organización industrial.

Así mismo, la revista está dirigida hacia economistas, profesionales en los negocios, comercio internacional y política pública, actuarios, administradores y profesionistas en matemática aplicada a las ciencias sociales.

En esta ocasión "PERSPECTIVAS: Revista de Análisis de Economía, Comercio y Negocios Internacionales", presenta a sus lectores el contenido del Volumen 12 (1) correspondiente al período enero – julio de 2018.

En este número se presentan cuatro trabajos con fuerte modelado matemático. El primero de ellos se refiere a una economía exportadora que hace uso de un recurso no renovable y desarrolla un modelo dinámico a los efectos de analizamos una política de gobierno para lograr un crecimiento económico sostenible a largo plazo. El segundo trabajo hace uso de las herramientas formales de la moderna teoría de finanzas, a los efectos de estimar los rendimientos de la deuda soberana en Uruguay en moneda nacional. El siguiente trabajo presenta un modelo de decisión dinámico, en el que un individuo debe decidir cómo y cuándo enfrentar las tareas más complejas. El cuarto trabajo que presentamos en este volumen hace referencia a la situación de desigualdad en Chile durante la década del noventa, presentando interesantes reflexiones sobre una economía emergente.

Entendemos que, el desarrollo de la teoría económica altamente formalizada es de gran interés para el crecimiento y el bienestar de los países del continente. La velocidad creciente con que los modelos más abstractos de la teoría económica logran una rápida aplicación a problemas empíricos muestra la necesidad de impulsar el estudio y la investigación en las diferentes áreas de la moderna teoría económica. Los eventos organizados por redes de trabajo tales como JOLATE (Jornadas Latino Americanas e Teoría Económica) y DGS (*Dynamics, Games and Science*) se convierten en un fuerte estímulo para el desarrollo de la investigación conjunta entre investigadores e instituciones latinoamericanas y europeas en las diferentes áreas de la moderna teoría económica y en particular de la economía matemática.

La organización periódica de eventos de este tipo y la difusión de los avances de los grupos de trabajo, contribuirá a enriquecer la interacción entre estudiantes, profesores e investigadores, y permitirá cumplir con una de las funciones sustantivas de la Universidad que consiste en la difusión de las ideas y pensamientos críticos que se dan en el seno del quehacer educativo universitario.

Es al servicio de la difusión y desarrollo de la investigación en la moderna teoría económica en sus diferentes aspectos que pretendemos poner nuestra publicación

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Fiscal policy in a non-renewable resource exporting economy

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Abstract

We present an exhaustible windfall profits model where the government exercises a fiscal sustainability framework that will allow an exporting resource-

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rich economy to transform into a technology based economy once the resource has been depleted.

Keywords: general equilibrium; natural resource rent; non-renewable revenue management; sustainable fiscal policy; taxation; windfall profits

JEL: D60, E2, H0, H21, I21, Q56.

1 Introduction

We construct a general equilibrium model for a resource-rich economy that includes three sectors: households, firms and a government. We analyze a government policy to bring about a sustainable long-run economic growth. That is, a public policy to transform depleting resource wealth into a time portfolio of other assets to support sustained economic growth and development, even after the resource is exhausted. Our model will allow us to analyze the fiscal and macroeconomic implication of saving/investment scaling up scenarios following the statement of Hartwick (1977) that the rent derived from resources should be reinvested to increase capital: infrastructure, physical capital, education and technological progress, necessary to guarantee a constant consumption in the course of time.

As results, we find the optimal rate of resource extraction and an optimal time profile of consumption, consumer's wealth, benefits and government domestic debt-spending.

The analysis is motivated by the work of Van-der-Ploeg and Venables (2010) who present a policy rule for a one sector economy, a welfare-maximization government that faces a temporary resource windfall by building a family of models according to three features of developing economies, namely: i) capital scarcity, ii) a set of instruments for a developing nation's government—small tax base and high premium on public funds—, and iii) a private sector that has no access to capital markets and thus, must live entirely from current wage income and government transfers. Next, Van-der-Ploeg and Venables (2010) derive the optimal time profile for consumption, foreign debt, public investment, tax and transfer policies. We differ from Van-der-Ploeg and Venables, (2010) in regard to the fact that ours is a general equilibrium model and it uses three sectors where each agent's objective function is maximized. We concentrate on the interaction between the three sectors/agents and the derivation of the optimal time profile of each one. In our model, we take the rate of technical progress as a substitute of the resource when it is depleted.

The structure of the paper is as follows: we present the model in Section 2. Section 3 presents the goods market equilibrium. Section 4 has our conclusions.

2 The Model

Time is continuous. We have three agents or sectors in our economy: households, firms and a government. Horizon is infinite but time is divided in two stages. During the first stage, the country has abundant non-renewable resource; during the second the resource has been depleted. Our objective is to maximize agents' objective function across both stages.

During the first stage, the government produces a non-renewable natural resource with no extraction costs and with a constant elasticity of demand for the resource. The firms' technology uses a production function, Y_t, which depends directly on physical capital, K_{y_t}, a positive quantity of the non-renewable resource, R_t—also called public capital—and labour, L_t. Households maximize their utility function after receiving a salary from the private sector, returns on their capital, τ, and transfers, T, from the non-renewable resource rent that the government obtains. We assume that τ and T are time invariant.

The key to the model is a fiscal policy that promotes a radical change from a non-renewable intensive economy to a technology based economy. In the first stage, the economy reinvests the windfall profits of the non-renewable resource in physical and human capital, and transforms so that when the resource has been fully exhausted, the economy no longer needs its income.

During the second stage, after the resource has been fully exhausted, the government makes up for the absence of the non-renewable resource income by charging firms and households income taxes and by issuing bonds. We recognize that going from having no fiscal obligations to having them may signify social turmoil. For example, the Mexican government decided to liberalize the price of gasolines in 2017—the price had been subsidized until then—and this was faced by severe civil opposition. The process to liberalization was announced in December 2016, the Mexican government announced that the price of gasolines would increase 20% on January 1st, 2017. This increase would be followed by two additional increases in February. Liberalization would begin on February 18, and it would begin by northern states. The process would end a year later. Once liberalization had been completed, prices would be set daily according to international prices[1]. This process was known as the *gasolinazo* and it generated numerous protests all over the country, gas station obstructions and raids. Civil unrest was such that on February 3, 2017, the government informed that it would refrain from the two increments it had announced for February, but liberalization would continue[2]. Our model aims at avoiding social turmoil by generating a

[1] See https://es.wikipedia.org/wiki/Protestas_por_el_precio_de_la_gasolina_en_M%C3%A9xico_de_2017

[2] See http://eleconomista.com.mx/finanzas-publicas/2017/02/03/no-habra-gasolinazo-manana-17-febrero

soft-landing.

2.1 The Non-Renewable Resource Production

We assume that the non-renewable resource has an initial stock, B_0, and its use over time is constrained to be equal to this fixed initial stock. That is, there are no discoveries of new non-renewable resource deposits.

Let Ω_t represent the amount of extraction of the non-renewable resource and B_t be the existing stock left at time t; we can write this constraint as:

$$B_t = B_0 - \int_0^t \Omega_s \;\; ds, \;\; B(0) = B_0 \tag{2.1}$$

This equation states that the stock remaining at time t, B_t, is equal to the magnitude of the initial stock, B_0, less the amount of the resource extracted, Ω_t, over the time interval $[0, t]$, where $\Omega_t \geq 0$.

By differentiating this equation with respect to time, we have

$$\dot{B}_t = -\Omega_t \tag{2.2}$$

$-\dot{B}_t$ is inversely proportional to Ω_t. The stock, B_t, converges to zero as more and more resource is extracted. Extraction continues until B_0 is fully exhausted.

From Ω_t, the government sells a quantity $\varepsilon_t\Omega_t$, $0 \leq \varepsilon_t \leq 1$, for all t, to domestic firms; and exports the rest $(1 - \varepsilon_t)\Omega_t$; ε is the amount that the domestic economy uses in the productive process. The government uses the proceeds as follows: i) it saves the flow in a sovereign wealth fund and uses the interests on these savings to transfer a quantity T to households; and, ii) the rest of the proceeds are used for current spending.

2.2 The Government

As mentioned, the government exports an amount $(1-\varepsilon_t)\Omega_t$ of the non renewable resource. We assume that the price of the resource is constant and equals to one $p_B = 1$. The difference, $\varepsilon_t\Omega_t$, is sold to firms at a subsidized price θp_B, $\theta \in (0,1)$, during the first stage; and at the international price p_B during the second stage. Let s_t stand for the flow of income that the government obtains from exporting the resource at time t,

$$s_t = (1-\varepsilon_t)\Omega_t p_B$$

which, given our assumption that $p_B = 1$ implies

$$s_t = (1-\varepsilon_t)\Omega_t. \tag{2.3}$$

We assume that from the very beginning $t = 0$, s_t is saved in a sovereign wealth fund, SWF, while investment capacity is built domestically [see Van-der-Ploeg and Venables (2010)]. These savings earn interest at a rate, r^*, where r^* represents the international time-invariant interest rate. The government saves to allow a desirable smooth consumption and expenditure across lifetime and to acquire a buffer against uncertain events.

Savings up to time t, S_t, are the sum of s_t over the interval $[0, t]$:

$$S_t = \int_0^t (1 - \varepsilon_s)\Omega_s ds \tag{2.4}$$

In the model we assume the *bird-in-hand* approach [Barnett and Ossowski (2002), Bjerkholt and Niculescu (2004)] under which resource revenue is placed in a SWF and only the interests on the savings are used to finance government expenditure (including transfers to households, T). That is, at time t, the government can only use interests on the current non-renewable resource income flow, $r^* s_t$, to finance a part of its current expenditures. The rest of government income is levied by a tax rate θ that domestic firms pay for the use of the non-renewable resource in its production process, θR_t.

Now, we consider the following indicator function, β_t.

$$\beta_t = \begin{cases} 1 & \text{if } (1 - \varepsilon_t)\Omega_t > 0 \\ 0 & \text{Otherwise.} \end{cases}$$

At each time t the government earns income by exporting the resource, that is, when $\beta_t = 1$; and the resource will be fully depleted when $\beta_t = 0$. As mentioned in the

Introduction, going from $\beta_t = 1$ to $\beta_t = 0$ will imply a traumatic change in the economy with potential social turmoil. The transition period can be smoothed if we allow for a third intermediate stage. Namely, when the non-renewable resource becomes scarce we let β_t converge to 0. The speed of convergence can be determined in order to minimize social unrest. This will all become clear when we study the effect of going from $\beta_t = 1$ to $\beta_t = 0$ on firms and households.

The economy ceases exporting if the resource is exhausted, $B_0 = 0$, or when the domestic resource consumption equals extraction, $\Omega_t = \varepsilon_t \Omega_t$. When exhaustibility takes place, we assume that the government replaces income by levying income taxes on firms $\delta \in (0, 1)$ and households $\psi \in (0, 1)$; and it begins to issue bonds, b_t, with a promise to pay periodic interest payments, rb_t, where r is the domestic time-invariant interest rate. It is this particular feature that may produce social turmoil: going from a resource wealthy economy that charges no income tax and subsidizes the use of the resource domestically; to a resource-deficient economy who sells the resource at international prices domestically and charges income taxes—the Mexican case mentioned in the Introduction—. To avoid that future generations be poorer than the present one, the government will continue transfers, T, to households after exhaustion.

Based on the previous discussion we now establish government expenditures, G_t

$$G_t = r^* s_t \beta_t + (1-\beta_t) r^* S_t + \theta R_t + (1-\beta_t)(\delta Y_t + \psi i_t + b_t) - T \tag{2.5}$$

The right hand side of the equation is the government's income. Where Y_t is firm income and i_t is household income. We multiply $(1-\beta_t)$ times $r^* S_t$ because after exhaustion, the accumulated savings will still generate returns. That is, we assume that government spending will switch from using interests on current income flow s_t to using interests on accumulated savings S_t.

Considering the government spending, its total net income, Gni_t, is:

$$Gni_t = r^* s_t \beta_t + (1-\beta_t) r^* S_t + (1-\beta_t)(\delta Y_t + \psi i_t + b_t) + \theta R_t - G_t - T \tag{2.6}$$

To be solvent the government should be able to meet the financial commitments; and the Non-Ponzi game condition must hold. Then the following dynamic budget constraint says that if the budget deficit is positive and there is essentially no financing by money creation, the government debt grows as:

$$\begin{aligned} &\dot{b}_t = (1-\beta_t) r b_t + T + G_t - (r^* s_t \beta_t + (1-\beta_t) r^* S_t) - \theta R_t - (1-\beta_t)(\delta Y_t + \psi i_t) \\ &\lim_{t\to\infty} b_t \exp(-rt) = 0 \\ &b(0) = b_0 \end{aligned} \tag{2.7}$$

2.3 Domestic firms

Grimaud et al. (2007) use a standard Cobb-Douglas production function to model the production of a homogeneous good. They regard energy as an input factor which is described by a function using non-renewable and renewable resources as well as human capital. In our model the amount of the non-renewable resource enters into the function production as just another factor of production.

We consider that the private sector produces a single good Y_t with a neoclassical production function which is linearly homogeneous, with constant returns to scale, and is non-decreasing and twice differentiable in each argument. This output is used for consumption as well as investment in capital and to increase technological progress.

$$Y_t = F(K_t, L_t, R_t) \tag{2.8}$$

Physical capital, K_t, is owned by the household and rented out to the private sector; τ is the time-invariant cost of capital. Households supply one unit of labor, L_t, inelastically and receive wages, w. The public capital R_t develops according to

$$R_t = \left[\beta_t(\varepsilon_t\Omega_t)^{\phi} + (1-\beta_t){A_t}^{\phi}\right]^{1/\phi} \tag{2.9}$$

Where $0 < \phi < 1$, is a substitute parameter, and A_t is the technological progress which enters into the production function as a substitute of the non-renewable resource when it is exhausted. A_t is a direct function of the amount of the final production dedicated to augmenting it. When A_t is introduced in the final good function we still assume it as a grade one homogeneous production function, as in Shell (1966).

The dynamics of A_t is

$$\dot{A}_t = \gamma \nu A_t \quad \text{where} \quad A_{t=0} = A_0 \tag{2.10}$$

The $\nu = s_{A_t} Y_t$, $0 < s_A < 1$, is the fraction of final output devoted to research activity and is also assumed time-invariant; $\gamma > 0$ is a productivity parameter of scientific research [see Romer (1990)], and A_t is the stock of technology at time t. For simplicity, we assume that technology does not decay overtime.

We suppose that all domestic prices—including wages, w, and the cost of capital, τ—, are constant and measured in terms of the output good which itself has a unit price, $p_Y = 1$. Then, the private sector revenue is $p_Y Y_t = Y_t$, and benefits, π_t, are:

$$\pi_t = Y_t \left(1 - (1 - \beta_t)\delta\right) - \tau K_t - w L_t \tag{2.11}$$

Where the δ is a private sector income tax.

Let us denote the dynamics of capital as

$$\begin{aligned} \dot{K}_t &= Y_t - \zeta K_t - (\nu + \theta) R_t \\ K(0) &= K_0 \ \text{ given} \end{aligned} \tag{2.12}$$

where ζ, $0 < \zeta < 1$, is the rate of capital depreciation, assumed constant; recall,

θ is the price or tax rate that domestic firms pay for the use of the non-renewable resource.

Given our assumptions on factor prices, the domestic firms' problem is to choose K_t, L_t and R_t so as to maximize the following objective function subject to (2.12)

$$\max \int_0^\infty \pi_t \exp(-rt)dt \tag{2.13}$$

Using a Cobb-Douglas function $Y_t = K_t^{\alpha_1} L_t^{\alpha_2} R_t^{1-\alpha_1-\alpha_2}; \quad 0 < \alpha_1 + \alpha_2 < 1$, we can write the current-value Hamiltonian as

$$H_c(L_t, K_t, R_t, \lambda_t) = \pi_t + \lambda_t \left(K_t^{\alpha_1} L_t^{\alpha_2} R_t^{1-\alpha_1-\alpha_2} - \zeta K_t - (\nu + \theta) R_t \right). \tag{2.14}$$

Where λ_t are the costate variables.

Taking first order conditions with respect to $\frac{\partial H_c}{\partial L_t} = 0$, $\frac{\partial H_c}{\partial R_t} = 0$ and $\dot{\lambda}_t = -\frac{\partial H_c}{\partial K_t} + \lambda_t r$, and solving for our control variables, we obtain their optimal values:

$$L_t^* = \left[\frac{w}{\alpha_2 K_t^{\alpha_1} R_t^{1-\alpha_1-\alpha_2} \left[1 - (1-\beta_t)\delta + \lambda_t\right]} \right]^{\frac{1}{\alpha_2 - 1}} \tag{2.15}$$

Then the price of the marginal product of labor, Y_{L_t}, equals wages,

$$\lambda_t Y_{L_t} \left[1 - (1-\beta_t)\delta + \lambda_t\right] = w \tag{2.16}$$

Deriving the $\frac{\partial H_c}{\partial K_t} = 0$, the cost of capital plus the value of the rate of depreciation equals the value of the marginal product of capital, Y_{K_t},

$$Y_{K_t}\left[1-(1-\beta_t)\delta+\lambda_t\right]=\tau+\lambda_t\zeta \tag{2.17}$$

Under constant returns to scale, in equilibrium the real domestic interest rate equals the value of the cost of capital minus the value of the rate of depreciation of physical capital

$$r=\tau-\zeta \tag{2.18}$$

The above equation represents the no profitable arbitrage condition. It shows the net income of capital owners per unit of capital. They earn the gross factor rewards τ at the same time, they experience a loss from depreciation. Net income therefore only amounts to r.

Solving for R_t gives

$$R_t^*=\left[\frac{\lambda_t\left(\nu+\theta\right)}{(1-\alpha_1-\alpha_2)K_t^{\alpha_1}L_t^{\alpha_2}\left[1-(1-\beta_t)\delta+\lambda_t\right]}\right]^{\frac{1}{-(\alpha_1+\alpha_2)}} \tag{2.19}$$

R_t^* is the optimal use of the non-renewable resource (the non-renewable resource while the resource exists and A_t afterwards) for domestic firms. The marginal product of R_t is

$$Y_{R_t}\left[1-(1-\beta_t)\delta+\lambda_t\right]=\lambda_t\left(\nu+\theta\right).$$

In equilibrium the marginal product of R_t equals the cost of technological progress minus the value of the resource capital.

$$Y_{R_t}\left[1-(1-\beta_t)\delta+\lambda_t\right]=\nu+\theta \tag{2.20}$$

If $\beta_t = 1$ then $Y_{R_t}(1+\lambda_t) = \lambda_t\theta$. That is, the marginal product of the use of the resource equals the price at which it is sold to firms. If $\beta_t = 0$ then $Y_{R_t}\left((1-\delta)+\lambda_t\right) = \lambda_t\nu$. Hence, the margial product of the use of the resource depends on the quantity of the resource dedicated to increasing technological progress.

For costate variables we have the following, $\dot{\lambda}_t = -\frac{\partial H_c}{\partial K_t} + \lambda_t r$, and given equation (2.17) and $r = \tau$, gives

$$\lambda_t = \lambda_0 \exp(-rt) \tag{2.21}$$

Given equation (2.17) we obtain the optimal capital

$$K_t^* = \left[\frac{\tau+\lambda_t\zeta}{\alpha_1 L_t^{\alpha_2} R_t^{1-\alpha_1-\alpha_2}\left[1-(1-\beta_t)\delta+\lambda_t\right]}\right]^{\frac{1}{\alpha_1-1}} \tag{2.22}$$

Equation (2.23) is the typical Hamiltonian equation of motion for K

$$\dot{K}_t = \frac{\partial H_c}{\partial \lambda_t} = Y_t - \zeta K_t - (\nu+\theta)\,R_t \tag{2.23}$$

Finally, domestic firms' optimal earnings before taxes are:

$$\pi_t^* = Y_t^* - \tau K_t^* - wL_t^*. \tag{2.24}$$

2.4 Households

We suppose that the economy is populated by a fixed number of infinitely-lived forward-looking consumers. We call them *dynasties*. Dynasties receive wages, w, returns on capital, τK_t, and government transfers, T. T is a fixed proportion of consumption, $\eta_t c_t$, $0 < \eta_t < 1$. Then household income i_t is:

$$i_t \left(1 - (1 - \beta_t)\psi\right) = wL_t + \tau K_t + T \tag{2.25}$$

we assume that government transfers influence households' ability to consume. Dynasties can make an estimation of their anticipated lifetime income given their salary, government transfers and resource exhaustion, and begin to save, because they expects that their long-run permanent income will be less that their current income, (Friedman, 1957). It is a way of avoiding countercyclical fluctuations of disposable income and of smoothing their consumption. After the natural resource has been exhausted households will invest in a government bond, b_t, that pays an interest rate, r, which is constant and free of default risk; and dynasties will begin to pay a tax rate, ψ, on their income, i_t.

Households consume and save, and savings at time t, b_t, can be loaned to the government.

When the dynamics of dynasties savings, b_t, equivalently their wealth, m_t, combine with a requirement of solvency, and the Non-Ponzi game condition, $\lim_{t\to\infty} m_t e^{-rt} = 0$, then we obtain the households' budget constraint as:

$$\begin{aligned} \dot{m}_t &= (\beta_t + (1-\beta_t)r)\, m_t - c_t + i_t(1-(1-\beta_t)\psi) \\ m(0) &= m_0 \end{aligned} \tag{2.26}$$

where $(1-\beta_t)m_t = b_t$, b_t are government bonds; and r multiplies m_t because household savings earn interests.

The household has perfect foresight and chooses a path $(c_\tau)_{\tau=t}^{\infty}$; that is, chooses a plan of consumption and savings which maximizes the discounted utility, U_t, subject to its budget constraint, (2.25), taking the time paths of the interest rate, government transfers, prices and the wage rate as given.

$$U_t = \int_0^\infty U(c_t)\exp(-\rho t)dt \tag{2.27}$$

where $\rho > 0$ is the rate of time preference. We assume that $U(c_t)' > 0$ and $U(c_t)'' < 0$. To avoid the possibility of corner solutions, we impose the assumption $\lim_{c\to 0} \dot{U}(c_t) = \infty$. We assume a constant relative risk aversion, CRRA, utility function, $\frac{c^{1-\sigma}-1}{1-\sigma}$, where $0 < \sigma < 0$ is the elasticity of intertemporal substitution; σ is the absolute elasticity of marginal utility w.r.t. consumption and it indicates the strength of the consumer's preference for consumption smoothing. Its inverse, $\frac{1}{\sigma}$, measures the instantaneous intertemporal elasticity of substitution in consumption, which in turn indicates the willingness to accept variation in consumption over time when interest rate changes.

We establish the current-value Hamiltonian as

$$H_c(c_t, m_t, \gamma_t) = \frac{c^{1-\sigma} - 1}{1-\sigma} + \gamma_t \left(\left(\beta_t + (1-\beta_t) r \right) m_t - c_t + i_t \left(1 - (1-\beta_t)\psi \right) \right) \quad (2.28)$$

First order conditions, $\frac{\partial H_c}{\partial c_t} = c_t^{-\sigma} - \gamma_t = 0$ give

$$c_t = \gamma_t^{-\frac{1}{\sigma}} \quad (2.29)$$

Calculating $\dot{\gamma}_t - \gamma_t \rho = -\frac{\partial H_c}{\partial m_t}$ we have

$$\gamma_t = \gamma_0 \exp\left(\rho - \left(\beta_t + (1-\beta_t) r \right) t \right). \quad (2.30)$$

Given equation (2.29) and (2.30) the household consumption is:

$$c_t = \gamma_0^{-\frac{1}{\sigma}} \exp\left(\frac{\left(\beta_t + (1-\beta_t) r - \rho \right) t}{\sigma} \right) \quad (2.31)$$

We derive γ_0 in order to find the optimal household consumption, (see the Appendix):

$$\gamma_0 = \left[\left(m_0 + \frac{i\left(1 - (1-\beta_t)\psi\right)}{r} \right) \frac{\rho - r(1-\sigma)}{\sigma} \right]^{-\sigma} \quad (2.32)$$

Given equation (2.32)

$$c_t^* = \left[\left(m_0 + \frac{i\,(1-(1-\beta_t)\psi)}{r} \right) \frac{\rho - r(1-\sigma)}{\sigma} \right] \exp\left(\frac{(\beta_t + (1-\beta_t)r - \rho)\,t}{\sigma} \right) \quad (2.33)$$

The timing of consumption (perfect smoothing) is not dependent on resource revenue flow (in spite the fact that initially it does depend on it) but on the initial wealth and the present value of income. Observe that consumption is bound to increase in the long run even after the non-renewable resource has been exhausted, which is a necessary condition for sustainable development.

Now, observe that from the optimal trajectory of consumption, determined in the equation (2.33), it follows that the existence of an interior solution also implies, the Keynes-Ramsey rule

$$\dot{c}_t = \frac{\beta_t + (1-\beta_t)r - \rho}{\sigma} \left[\left(m_0 + \frac{i\,(1-(1-\beta_t)\psi)}{r} \right) \frac{\rho - r(1-\sigma)}{\sigma} \right] \exp\left(\frac{(\beta_t + (1-\beta_t)r - \rho)\,t}{\sigma} \right)$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\big(\beta_t + (1-\beta_t)r - \rho\big) \quad (2.34)$$

Equation (2.34) says that an optimal consumption plan is characterized as follows: the household will completely smooth consumption over time if interes rate is greater than the rate of time preference. In this case the household will have to accept a relatively low level of current consumption with the purpose of enjoying higher consumption in the future.

We now state the typical Hamiltonian equation of motion for m_t

$$\dot{m}_t = \frac{\partial H_c}{\partial \lambda_{h_t}} = (\beta_t + (1-\beta_t)r)\, m_t - c_t + i_t(1-(1-\beta_t)\psi) \tag{2.35}$$

2.5 Government Problem Specification

The government maximizes the objective function at time t, U_t, adjusting resource exporting, transfers and taxes so that the budget constraint, (2.7), is satisfied at any t, taking prices and r^* as given

$$\max U_t = \int_0^\infty U(c_t)e^{-\rho t}dt \tag{2.36}$$

We assume that U_t is such that $U(c_t)' > 0$ and $U(c_t)'' < 0$. By using the following utility function

$$U(c_t) = \frac{c^{1-\sigma} + \Theta G^{1-\sigma}}{1-\sigma}, \quad \rho > 0$$

where $\Theta > 0$ is the weight given to government expenditures, then we can write the Hamiltonian of this problem as

$$\begin{aligned} H_c(\Omega, \varepsilon, \eta, G_t, \mu_t) &= \frac{c^{1-\sigma} + \Theta G^{1-\sigma}}{1-\sigma} \\ &+ \mu_t \Big((1-\beta_t) r \ b_t + T + G_t - r^* s_t \beta_t - (1-\beta_t) r^* S_t - \theta R_t - (1-\beta_t)(\delta Y_t + \psi i_t) \Big) \\ &= \frac{c^{1-\sigma} + \Theta G^{1-\sigma}}{1-\sigma} \\ &+ \mu_t \Big(T + G_t - r^* s_t \beta_t - \theta R_t - (1-\beta_t) \big[(\delta Y_t + \psi i_t) + r^* S_t - r \ b_t \big] \Big) \end{aligned} \tag{2.37}$$

Let $T = \eta_t c_t$, taking (2.33) into account, we solve for η_t as

$$\eta_t^* = -\frac{\left\{ \left[\left(m_0 + \frac{i(1-(1-\beta_t)\psi)}{r} \right) \frac{\rho - r(1-\sigma)}{\sigma} \right] \exp\left(\frac{(\beta_t + (1-\beta_t) r - \rho) t}{\sigma} \right) \right\}^{-\sigma}}{\mu_t} \tag{2.38}$$

For $\frac{\partial H_t}{\partial G_t} = 0$ the result is

$$G_t = -(\frac{\mu_t}{\Theta})^{-}\frac{1}{\sigma} \tag{2.39}$$

$$\mu_t = \mu_0 \exp((\rho - (1-\beta_t) r) t) \tag{2.40}$$

$\frac{\partial H_c}{\partial \Omega_t} = 0$ and given equation (2.4) if $\beta_t = 1$, $R_t = \varepsilon_t \beta_t \Omega_t$, then $\frac{\partial R_t}{\partial \Omega_t} = \varepsilon_t \beta_t$

The optimal rate, ε_t, of the use of the resource in the domestic economy is

$$\varepsilon_t = \frac{r^*}{r^* - \theta p_B} \tag{2.41}$$

recall, θ is the price or tax rate that domestic firms pay for the use of the non-renewable resource. Equation (2.41) is our equilibrium version of the Hotelling rule: a no-arbitrage condition between investing in the resource (leaving it in the ground) and investing in financial assets to receive r^*. Hotelling (1931) showed that the optimal path of extraction of a non-renewable resource, in a competitive environment, corresponds to the situation where the resource extraction rate grows at a rate equal to the international interest rate, r^*.

Equating (2.9) and (2.19), and if $\beta_t = 1$, then $R_t = \varepsilon_t \Omega_t$ and equation (2.19) becomes

$$R_t^* = \left[\frac{\lambda_t \theta}{1 - \alpha_1 - \alpha_2 K_t^{\alpha_1} L_t^{\alpha_2}(1+\lambda_t)}\right]^{\frac{1}{-(\alpha_1+\alpha_2)}}.$$

Now we derive the efficient rate of extraction Ω_t as

$$\Omega_t^* = \frac{1}{\varepsilon_t}\left[\frac{\lambda_t \theta}{(1 - \alpha_1 - \alpha_2) K_t^{\alpha_1} L_t^{\alpha_2}\left[1 - (1-\beta_t)\delta + \lambda_t\right]}\right]^{\frac{1}{-(\alpha_1+\alpha_2)}} \tag{2.42}$$

If, on the contrary, $\beta_t = 0$, then $R_t = A_t$ and equation (2.18) becomes

$$A_t^* = \left[\frac{\lambda_t \nu}{(1 - \alpha_1 - \alpha_2) K_t^{\alpha_1} L_t^{\alpha_2}\left[(1-\delta) + \lambda_t\right]}\right]^{\frac{1}{-(\alpha_1+\alpha_2)}}. \tag{2.43}$$

3 Goods market equilibrium

The goods market equilibrium requires that supply equals demand, $Y_t = C_t + I_t$, where demand is given by consumption plus gross investment. The change in the capital stock $\dot{K}_t$ is given by gross investment I_t minus depreciation ζK_t, $\zeta \in (0,1)$ is the depreciation rate,

$$\dot{K}_t = I_t - \zeta K_t.$$

Replacing gross investment with the goods market equilibrium, we obtain the dynamics of capital as:

$$\dot{K}_t = Y_t - C_t - \zeta K_t \tag{3.44}$$

Given Euler‘s Theorem $Y_t = \frac{\partial Y_t}{\partial K_t} + \frac{\partial Y_t}{\partial L_t} + \frac{\partial Y_t}{\partial R_t}$; and according to equations (2.16), (2.18) and (2.20) the dynamics of capital is given by:

$$\dot{K}_t = rK_t + wL_t + (\nu + \zeta p_B)R_t - C_t \tag{3.45}$$

Recall we have assumed that $p_B = 1$. This equation reflects a goods market equilibrium capital dynamics. Thus, the capital of this economy depends on the household's income plus resources dedicated to increase technological progress plus the price of the resource minus aggregate consumption.

4 Conclusions

In this model we have analyzed a government policy to bring about a sustainable long-run economic growth. That is, we have developed a model that analyzes the fiscal and macroeconomic implications of saving/investment scaling up scenarios. By promoting non-renewable resource efficiency we find the optimal rate of resource extraction and the optimal time profile of consumption, benefits and government domestic debt-spending.

Also, the model has reached, through optimal derivation, the sustainable consumption and production by promoting resource efficiency, sustainable consumption, and providing accessibility of the natural resource to the whole economy. This implementation helps to achieve an overall development plan; it reduces future economic, environmental and social costs; strengthens economic competitiveness; and reduces poverty.

Appendix

$$c_t = \gamma_0^{-}\frac{1}{\sigma}e^{\frac{((\beta_t+(1-\beta_t)r)-\rho)t}{\sigma}}$$

$$c_t e^{-rt} = ((\beta_t + (1-\beta_t)r))m_t - \dot{m}_t + i_t(1-(1-\beta_t)\psi)$$

Multiplying by e^{-rt} both sides and integrating we get:

$$\int_0^\infty c_t e^{-rt}dt = \int_0^\infty (((\beta_t + (1-\beta_t)r))m_t - \dot{m}_t)e^{-rt}dt + \int_0^\infty i_t(1-(1-\beta_t)\psi)e^{-rt}dt$$

Solving the first integral in RHS by integration by parts, assuming $\lim_{t\to\infty} m_t e^{-rt} =$

0 and $i_t = i$ constant, we obtain:

$$\int_0^\infty c_t e^{-rt} dt = m_0 - \int_0^\infty \dot{m}_t e^{-rt} dt + \int_0^\infty \dot{m}_t e^{-rt} dt + i(1-(1-\beta_t)\psi) \int_0^\infty e^{-rt} dt$$

$$\int_0^\infty c_t e^{-rt} dt = m_0 + i(1-(1-\beta_t)\psi) \int_0^\infty e^{-rt} dt$$

$$\int_0^\infty \gamma_0^{-} \frac{1}{\sigma} e^{\frac{((\beta_t + (1-\beta_t)r)-\rho)t}{\sigma}} e^{-rt} dt = m_0 + i(1-(1-\beta_t)\psi) \int_0^\infty e^{-rt} dt$$

$$\gamma_0^{-} \frac{1}{\sigma} \int_0^\infty e^{\frac{-(\rho - r(1-\sigma))t}{\sigma}} dt = m_0 - \frac{i(1-(1-\beta_t)\psi)}{r}$$

$$\gamma_0 = \left[\left(m_0 + \frac{i(1-(1-\beta_t)\psi))}{r} \right) \frac{\rho - r(1-\sigma)}{\sigma} \right]^{-\sigma}$$

References

[1] Barnett S., Ossowski R. (2002) Operational aspects of fiscal policy in oil-producing countries. International Monetary Fund Working Paper WP/02/177

[2] Bjerkholt O., Niculescu I. (2004) Fiscal rules for economies with nonrenewable resources: Norway and Venezuela. In: Kopits G. (ed) Rules-Based Fiscal Policy in Emerging Markets: Background, Analysis and Prospects. Palgrave Macmillan UK

[3] Friedman M. (1957) A theory of the consumption function. The permanent income hypothesis. Princeton University Press.

[4] Grimaud A., Lafforgue G., Magne B. (2007) Innovation markets in the policy appraisal of climate change mitigation. *LERNA Working Papers* (07.12.233) via DIALOG: http://EconPapers.repec.org/RePEc:ler:wpaper:07.12.233 Cited March 23, 2017.

[5] Hartwick, J. M. (1977) Intergenerational equity and investing rents from exhaustible resource rent. American Economic Review 66: 972-974.

[6] Hotelling M. (1931) The economics of exhaustible resources. Journal of political Economy 39: 137-175.

[7] Romer P.M. (1990) Endogenous technological change. The Journal of Political Economy 98(5): S71-S102.

[8] Shell K. (1966) Toward a theory of inventive activity and capital accumulation. American Economic Review 56(1/2): 62-68.

[9] Van-der-Ploeg F., Venables A.J. (2010) Harnessing windfall revenues: Optimal policies for resource-rich developing economies. The Economic Journal 121: 1-30