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Social contracts, free riders and utilities

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Social contracts, free riders and utilities

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This paper argues that the existence of selfish behavior in a given society does not depend only of individual preferences but also on the conditions prevailing in each society. Moreover the selfish behavior is not necessarily the best option, even when individuals are maximizers of their welfare.

1 Introduction

As it is well known the Coase theorem [Coase, R.] argue that, even in the presence of externalities, economic agents should still be able to ensure a Pareto-efficient outcome without government intervention provided that there are no constraints on their ability to bargain and contract. The argument is straightforward: if a prospective allocation is inefficient, agents will have the incentive to bargain their way to a Pareto improvement. Thus, even if markets themselves fail, Coasians hold that there is still a case for laissez-faire.

As is shown in [Maskin E, Baliga S.], the Coasian position depends, on the requirement that any externality present be excludable, in the sense that the agent giving rise to it has control over who is and who is not affected by it. A pure public good, which, once created, will be enjoyed by everybody, constitutes the classic example of a nonexcludable externality.

We consider an extension of the Maskin-Baliga model and we show that the possibility to bargain a contract to obtain a Pareto-efficient allocation, without the intervention of a central authority depends on the utilities over the good creating externalities during consumption. As a particular case we obtain the Maskin-Baliga model where there is no way to obtain such kind of contract without the participation of the central authority.

The rest of the paper is organized as follows. In (2) we introduce the model. The definitions of allocation social and individualistic allocations is given in section (3) and we discuss the relationships between individuals utilities and incentives to the selfish behavior. We consider a possible generalization of the concept of best response. Next, we introduce in section (4) a dynamical system characterizing the evolution of the communities with respect to the environmental care. Finally, in the last section, we give some conclusions.

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2 The Model

There are N players or agents, indexed by $j \in 1, 2, \dots, N$, and a set of social choices (or social decisions) Y with generic element y . Agents have preferences over the social choices, and these depend on their preference parameters or types. Agent j of type θ_j has a utility function $U_j(y, \theta_j)$. A choice y is (ex-post) Pareto-efficient for preference profile θ if there exists no other decision y' such that, for all $i \in \{1, \dots, N\}$,

$$U_i(y', \theta_i) \geq U_i(y, \theta_i)$$

with strict inequality for some i . By $\theta = (\theta_1, \dots, \theta_N) \in \Theta$ we represent a preference profile or state, where $\Theta = \prod_{i=1}^N \Theta_i$ is the set of possible states.

As in [Maskin E, Baliga S.] we consider an example based on contamination, where N communities (labeled $i \in \{1, \dots, N\}$) would like to reduce their aggregate emission of pollution. Suppose that the gross benefit to community j of a pollution reduction r is $\theta_j r^\alpha$ $\alpha \in (0, 1)$ and $\theta_j \in [a, b]$, all and that the cost per unit of reduction is 1. We consider that all these θ_j are different. If r_j is the reduction of pollution by community j , $r = \sum_{i=1}^N \theta_i$.

As we shall show for some values of α we obtain that the Coasian bargaining will not lead to Pareto-efficient pollution reduction, however it is possible to obtain this efficient allocation is α is bigger enough.

The net profit for each community will be

$$u_j(y, \theta_j) = \theta_j r^\alpha - r_j$$

For each level r of pollution reduction, a Pareto-efficient allocation can be obtained as the result of the maximization program:

$$\begin{aligned} \max_{r_j} \sum_{i=1}^N [\theta_i r^\alpha - r_i] \\ \sum_{i=1}^N r_i = r. \end{aligned} \tag{1}$$

or equivalently

$$\begin{aligned} \max_{r_j} \sum_{i=1}^N \theta_i r^\alpha - r \\ \sum_{i=1}^n r_i = r. \end{aligned} \tag{2}$$

3 Social and individual optimal allocations

A benevolent social planner look for this program and for the maximum value of r . This maximum correspond to

$$r^*(\theta_1, \dots, \theta_N, \alpha) = \left(\alpha \sum_{i=1}^N \theta_i \right)^{\frac{1}{1-\alpha}}. \quad (3)$$

Note that this social maximum value of the reduction, increase with $\alpha \in (0,1)$, because $\frac{\partial r^*(\theta_1, \dots, \theta_N, \alpha)}{\partial \alpha} > 0 \quad \forall \alpha \in (0,1)$ and with each $\theta_j, j \in \{1, \dots, N\}$.

3.1 Solidary agent and opportunistic agent

Our next question is about if this optimal value of reduction of the contamination, can be reached as result of the free action of the communities. Since in this case, each community wish to maximize her utility function, the total reduction of the contamination will be equal to the sum of the solution of the following N maximization problems:

$$\theta_j \left(r_j + \sum_{i \neq j} r_i(\theta_i) \right)^\alpha - r_j, \quad \forall j \in \{1, \dots, N\}. \quad (4)$$

The result of these program is:

$$r_j^{**}(\theta_j) = \begin{cases} \left(\alpha \theta_j \right)^{\frac{1}{1-\alpha}} & \theta_j = \max \{ \theta_1, \dots, \theta_N \} \\ 0 & \text{other case} \end{cases} \quad (5)$$

So in this case the maxima in the reduction of contamination is

$$r^{**}(\theta_1, \dots, \theta_N, \alpha) = \left(\alpha \theta_j \right)^{\frac{1}{1-\alpha}}, \text{ where } \theta_j = \max \{ \theta_1, \dots, \theta_N \}.$$

Clearly

$$r^{**}(\theta_1, \dots, \theta_N, \alpha) < r^*(\theta_1, \dots, \theta_N, \alpha).$$

Now, suppose that the communities attempt to negotiate the Pareto-efficient reduction by, say, agreeing to share the costs in proportion to their benefits, by, say, agreeing to share the costs in proportion to their benefits. That is, community j will pay a cost equal to

$$\theta_j \left[\alpha \left(\sum_{i=1}^N \theta_i \right)^{\frac{1}{1-\alpha}} \right]^\alpha.$$

So, the net payoff is

$$\theta_j \left[\left(\alpha \sum_{i=1}^N \theta_i \right)^{\frac{1}{1-\alpha}} \right]^\alpha - \theta_j \left[\alpha \left(\sum_{i=1}^N \theta_i \right)^\alpha \right]^{\frac{1}{1-\alpha}} = \theta_j \left[\alpha \sum_{i=1}^N \theta_i \right]^{\frac{\alpha}{1-\alpha}} (1-\alpha) \quad (6)$$

The community j prefer to be free rider if and only if the following inequality holds:

$$\theta_j \left[\alpha \sum_{i \neq j}^N \theta_i \right]^{\frac{\alpha}{1-\alpha}} > \theta_j \left[\alpha \sum_{i=1}^N \theta_i \right]^{\frac{\alpha}{1-\alpha}} (1-\alpha). \quad (7)$$

Consider N communities characterized by $\theta = (\theta_1, \theta_2, \dots, \theta_N)$, suppose that $\theta_i \neq \theta_j$, if $i \neq j \in \{1, \dots, N\}$. And let $\beta = (1-\alpha)^{\frac{1-\alpha}{\alpha}}$. Then the following theorem holds.

Theorem 1 *The community j prefers to be free rider if and only if*

$$\sum_{i \neq j} \theta_i > \left[\frac{\beta}{1-\beta} \right] \theta_j. \quad (8)$$

Proof: From (7) it follows that the community j prefer to be free rider if and only if:

$$\sum_{i \neq j} \theta_i > \left(\sum_{i \neq j} \theta_i + \theta_j \right) (1-\alpha)^{\frac{1-\alpha}{\alpha}}. \quad (9)$$

Since $\beta = (1-\alpha)^{\frac{1-\alpha}{\alpha}}$, after some algebraica manipulations it follows that

$$\sum_{i \neq j} \theta_i > \left[\frac{\beta}{1-\beta} \right] \theta_j. \bullet$$

Note for each $\alpha \in (0,1)$ there is only one value of β . So we can consider the function $\beta: (0,1) \rightarrow R$ defined by

$$\beta(\alpha) = (1-\alpha)^{\frac{1-\alpha}{\alpha}}.$$

This is an increasing function of α , see figure (1), verifying in addition that:

$$\lim_{\alpha \rightarrow 0} \beta(\alpha) = \frac{1}{e}$$

$$\lim_{\alpha \rightarrow 1} \beta(\alpha) = 1.$$

Figure 1: $\beta(\alpha) = (1-\alpha)^{\frac{1-\alpha}{\alpha}}$

Consider now the function $F : \left(\frac{1}{e}, 1\right) \rightarrow \mathbb{R}$ where $F(\beta) = \frac{\beta}{1-\beta}$. This an increasing function creciente of β . Note that $F'(\beta) = \frac{1}{(1-\beta)^2} > 0$. The next two conditions are also verified by F

$$\lim_{\beta \rightarrow \frac{1}{e}^+} F(\beta) = \frac{1}{e-1}$$

and

$$\lim_{\beta \rightarrow 1^-} F(\beta) = \infty.$$

The composite function $F \circ \beta(\alpha)$ is well defined in the open interval $(0,1)$. It is an increasing function, verifying that $F \circ \beta\left(\frac{1}{2}\right) = 1/2$.

For $\alpha = \frac{1}{2}$ we recover the example of Maskin and Baliga, see [Maskin E, Baliga S].

The next corollary follows.

Corollary 1 *If $\alpha = \frac{1}{2}$ all communities, except for the one with a greater interest in clean air, i.e, $\bar{j} : \theta_j = \max\{\theta_1, \dots, \theta_N\}$, prefer to be free-rider.*

Proof: If $\alpha = \frac{1}{2}$ then $\beta = \frac{1}{2}$ then condition for free rider, given in theorem (1) equation (1) can be written as $\sum_{i \neq j} \theta_i > \theta_j$. This condition is verified by all communities, with the possible exception of that with greater taste for clean air. •

Note that this corollary establishes that all communities with the possible exception of that with greater taste for clean air prefer to be free rider, and moreover, no matter how many communities have dropped the agreement before, this decision does not change. Several experimental works show that heterogeneity in preferences makes voluntary cooperation fragile, see for instance [Fischbacher, U, and Gächter, S]. The theoretical result obtained for $\alpha = \frac{1}{2}$ seems to confirm the experimental result. But it can not be generalizad because, it is possible that a cooperative behavior be more rewarding than the individualistic one, this depends on the strategic environments, and on the characteristics of individual preferences. In particular, in our model, if α is big enough, every community prefers cooperation, that is their prefer to maintain the agreement.

Corollary 2 *If α is big enough then every community prefer to participate in the agreement.*

proof: If $\alpha \rightarrow 1$ then $\frac{\beta}{1-\beta} \rightarrow \infty$. So, if $1 > \alpha > \alpha_0 > 0$ then $\frac{1-\beta}{\beta} \sum_{i \neq 1} \theta_i < \theta_1$, or equivalently $\frac{1-\beta}{\beta} \sum_{i \neq j} \theta_i < \theta_j \quad \forall j$. •

Example 1 . In this example a case in which all communities prefer to participate in the agreement shows.

Let $N = 3$, and $\theta_1 = 0,2$, $\theta_2 = 0,3$, $\theta_3 = 0,4$ y $\alpha = 0,99$

Then $\beta \approx 0.9545$ and $\frac{1-\beta}{\beta} \approx 0.04766$ it follows that the condition () is not verified, i.e.;

- $0,04766(0,2+0,3) < 0.4$.
- $0,04766(0,2+0,4) < 0.3$.
- $0,04766(0,3+0,4) < 0.2$

Example 2 . For $\alpha = .9$ and the same dates than in example (1), we have that only community 1 prefers to be free rider. We obtain that $\beta \approx 0.7742$ and $\frac{1-\beta}{\beta} \approx 0.2915$. We obtain the following inequalities:

- $0,2915(0,3+0,4) > 0.2$.
- $0,2915(0,2+0,4) < 0.3$.
- $0,2915(0,2+0,3) < 0.4$.

Example 3 Let $\alpha = 0.89$ then $\beta \approx 0.7612$, $\frac{1-\beta}{\beta} \approx 0.3$,

$\theta_1 = 0.2$, $\theta_2 = 0.3$, $\theta_3 = 0.4$, $\theta_4 = 0.5$, then we have the following inequalities:

- $0.3(0,3+0.4+0.5) > 0.2$
- $0.3(0,2+0.4+0.5) > 0.3$ but observe that
- $0.3(0.4+0.5) < 0.3$

So, communities 1, and 2 prefer to be free riders but only if all the others are cooperating. But, if community 1 leaves the agreement then, community 2 prefers to remain.

Theorem 2 Suppose that j^* verify the condition $\sum_{i \neq j^*} \theta_i < \left[\frac{\beta}{1-\beta} \right] \theta_{j^*}$ and that the community j' check reverse inequality, i.e; j' prefers to be free rider. Then all community

j $\theta_j > \theta_{j^*}$ prefer solidarity, while all community verifying $\theta_j < \theta_{j^*}$ prefer to be free rider.

Proof: It is straightforward from condition (1). Certainly, it is enough to verify that

$$\sum_{i \neq j^*} \theta_i > \sum_{i \neq j'} \theta_i \quad \forall j : \theta_j > \theta_{j^*}, \text{ and that } \sum_{i \neq j'} \theta_i < \sum_{i \neq j^*} \theta_i \quad \forall j : \theta_j < \theta_{j^*}.$$

3.2 An intuition in the value of α for cooperation

Consider that every θ_i is in a neighborhood of a given value z . Then,

$$r^*(\theta_1, \dots, \theta_N) = (\alpha N z)^{\frac{1}{1-\alpha}}$$

$$r^{**}(\theta_1, \dots, \theta_n) = (\alpha z)^{\frac{1}{1-\alpha}}$$

So,

$$\begin{aligned} r^*(\theta_1, \dots, \theta_N) - r^{**}(\theta_1, \dots, \theta_n) &= (\alpha N z)^{\frac{1}{1-\alpha}} - (\alpha z)^{\frac{1}{1-\alpha}} = \\ &= (\alpha z)^{\frac{1}{1-\alpha}} \left[N^{\frac{1}{1-\alpha}} - 1 \right]. \end{aligned}$$

The social optimum reduction differs from the individual aggregate reduction by a factor equal to $N^{\frac{1}{1-\alpha}}$.

If $\alpha \rightarrow 0$ this difference become small, meaning that if a community choose to be free rider, the difference in the quality of the aire does not change to much after this community leaves the agreement. But in so far α increases, this difference increases and so, the incentive to be free rider is countered by this effect.

3.3 The current behavior of the others it is not enough to explain the current choice

Note that if $\alpha > \frac{1}{2}$ even is the community j prefers to be free rider, i.e: if the condition (1) is verified, it is not necessarily true that this community prefers to leave the agreement if previously, another community j' left the agreement. Because

$$\sum_{i \neq j} \theta_i > \left[\frac{\beta}{1-\beta} \right] \theta_j \text{ does not imply}$$

$$\sum_{\substack{i \neq j \\ i \neq j'}} \theta_i > \left[\frac{\beta}{1-\beta} \right] \theta_j.$$

See example (3). Suppose that there is a large number of different communities, each characterized by $\theta_i \in [a, b]$, and that each one must choose at a given time, between to be free rider (FR) or solidary (S) (i.e, to do some solidary effort to clean air). Suppose in

addition that every community such that $a \leq \theta_i \leq d < b$ prefers to be free rider, and that every community such that $d \leq \theta_i \leq b$ prefers to be solidary. In the sense that $u_i(FR_i, S_{-i}) > u_i(S_i, S_{-i})$ for all i such that $\theta_i \in [a, d]$ and $u_i(S_i, S_{-i}) > u_i(FR_i, S_{-i})$ for all i such that $\theta_i \in [d, b]$. By S_{-i} we represent that every community except the i -th is following a solidary behavior.

However when each community faces the choice between to be FR or S, not only looks for the current situation, because the choice of others communities can affect the future payoff associate with the available strategies. So, a key point is that, the available information, over the actual and future situations plays a crucial role.

Suppose that community j prefers to be free rider, i.e $u_j(FR_j, S_{-j}) > U_j(S_j, S_{-j})$ but is not necessarily true that $u_j(FR_j, S_{-[a,c]}) > U_j(S_j, S_{-[a,c]})$, where by $S_{-[a,c]}$ we symbolize that all communities with $\theta_i \in [a, c]$ left the agreement. In this case, it is not necessarily true, that the j -th community still prefers to be free rider if a significant number of communities previously have chose this alternative.

Note that the current payoff of each individual strategic choice depends on what the others are doing, the classical theory considers that is taking account this fact, that every player chooses his own strategy. But in our case, each community makes its own strategic choice, taking into account not only what others are doing at the moment, but also considers how the choice of others affect the future performance of each of their currently available choices. This modify the content of the best response because, classically to define this concept, is only considered what others are doing at the moment, but not how the current choices of others affects the future returns associated with each available strategy.

As we already show, if α is bigger enough, every community prefers to cooperate. But unfortunately, this is not the general case. In many cases, there are incentives for the selfish behavior, in particular if α is small. In such cases, as we previously claimed, the future behavior of the communities may depend on their beliefs on what others will do. For such cases, a question whose answer is of great importance is, how a community anticipates the behavior of others. The problem is that the future return associated with each strategy depends more on what others will do, than of what they are currently doing.

Whatever it seems natural to assume that the incentives to be free riders decrease proportionally to the quantity of free riders existing in a given time, and increase proportionally to the quantity of cooperative individual existing in a given society. This assumption are natural, because as the percentage of free riders increase, the quality of the environment worsens, thus the incentives that new communities behave as free riders decreases. The opposite happens if the number of cooperative or supportive individuals increases. In this case an individual deviates from the cooperative behavior will not affect largely the quality of the environment, and who deviate can improve. But this situation is not maintained over time. In the measure that the percentage of individuals who fail to cooperate increases, the individual behavior become relevant for the environmental quality. At some point, the damage that a new free rider can cause to the environment, affects the individual welfare, in such way that the potential profits of a selfish behavior is

completely countered.

A new question is if there exists a threshold, after that it is not possible to recuperate the previous situation. It may be that after a certain number of communities choose to be free riders, environmental degradation is so large that there are not incentives to new free riders. The problem that arises is if after this moment the environmental degradation is or not reversible.

4 A dynamics for the environmental care

Let us symbolize by $x_{fr}(t)$ the porcentaje of communities following a free rider behavior in time t and let $x_{co}(t)$ the percentage of community that follows in time t a cooperative behavior. For all t the identity $x_{fr}(t) + x_{co}(t) = 1$ holds.

The following system of differential equations corresponds to our assumption about the dynamical relations between free riders and cooperative individuals.

$$\begin{aligned}\dot{x}_{fr} &= (-A + Bx_{co})x_{fr} \\ \dot{x}_{co} &= -\dot{x}_{fr}\end{aligned}\tag{10}$$

$$x_{fr}(0) = x_0, x_{co}(0) = 1 - x_0$$

Where A and B are positive real numbers. Since $x_{co}(t) = 1 - x_{fr}(t)$ we obtain the differential equation:

$$\dot{x}_{fr} = (-A + B)x_{fr} - Bx_{fr}^2\tag{11}$$

$$x_{fr}(t_0) = x_0$$

∴

$$x_{fr}(t) = \frac{(B-A)x_0}{Bx_0 + (B-A + Bx_0)e^{(B-A)t}}$$

There are two different dynamical equilibria

$$x_{fr}^e = 0 \rightarrow x_{co}^e = 1, \text{ and}\tag{12}$$

$$\hat{x}_{fr}^e = \frac{B-A}{-B} \rightarrow \hat{x}_{co}^e = \frac{-2B+A}{-B}.$$

The values that the parameters A and B will depend on the α . We can consider that A and B are decreasing functions of α because as α increase the incentives to be free riders decrease. Note that

- if $B - A < 0$ then the equilibrium the equilibrium $(\hat{x}_{fr}^e, \hat{x}_{co}^e)$ is asymptotically globally stable, and the evolution is toward a society where there are both types of communities, free riders and cooperatives. The equilibrium $(0,1)$ is unstable.

- if $B - A > 0$ then the situation is the inverse. The society is evolving toward $x_{fr}^e = 0 \Rightarrow x_{co}^e = 1$, i.e free riders tends to disappear. Note that, this result is strongly related with our previous assumptions. The difference $(B - A)$ increase with α , and if α is bigger enough then all communities prefer to be cooperative.

5 Conclusions

As we have shown in this paper, the existence of free riders, i.e; individuals who do not contribute to a collective project but still benefit from it, depends on the heterogeneity of the utility functions of the individuals, communities or groups, involving in the consumption of a non excludable good creating externalities. Generally a set of free riders and a set of cooperative individuals, i.e, individuals coordinating their actions to reach a common goal and then sharing the resulting benefits, coexist. The fact that an individual participates in either group is not necessarily an idiosyncratic question, (not necessarily depends on the own preferences) depends on occasions, the time when the decision to behave in a way or another is taken and in particular, it depends on the current distribution at the time in which each individual should take a decision. Moreover, given the individual preferences, under different distributions of the society in free riders and cooperatives, the same individual (or community) can made different choices.

Furthermore, although the literature is abundant in describing the mechanisms to punish selfish behavior, it is unclear whether or not this is advantageous. Under certain conditions compel individuals to collaborate on a project that requires coordination and sacrifice, can be counterproductive to the project itself. Note that the selfish behavior does not depend exclusively on the individual's preferences, but also on the preferences of the others members of the society. An individual may prefer to be free rider if many people are following cooperative behavior, but otherwise, it may happen that the same individual with the same preferences, decides to cooperate. In particular, it may happen that an individual, given the current behavior of their peers, prefers to be free rider, but if he anticipates the future behavior of others, decides to remain as a partner of the collective project.

For future works we leaves the question about the existence of a threshold for the reversibility of the environmental damage.

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